Asymmetric Access to Consumer Data and Softened Competition*

Gregory Dannay†

October 30, 2026

Abstract

Asymmetric access to consumer data is common in digital markets. I study a sequential Bertrand duopoly in which one firm has richer data than its competitor and can commit to a segmentation policy for personalized pricing, while its competitor sets a uniform price. In equilibrium, the optimal information structure coarsens consumer data to soften price competition. Under a monotone loyalty condition, every consumer is worse off than both when the informed firm sets a uniform price and when it fully personalizes its prices. Without this condition, consumers remain worse off in aggregate (i) than under full personalization, and (ii) than under uniform pricing in fully covered market outcomes.

Keywords: Market Segmentation, Competition, Price Discrimination.

JEL Codes: D43, D83, L13.

*I thank Laurent Mathevet, Giacomo Calzolari, and Eduardo Perez-Richet for their guidance. I also thank Özlem Bedre-Defolie, Zeinab Aboutalebi, Nikhil Vellodi, Balazs Szentes, Jidong Zhou, Matthew Mitchell, Jeanne Hagenbach, Franz Ostrizek, Alex Smolin, Victor Augias, and Zvika Neeman for insightful discussions and comments at various stages of the project, as well as participants at various seminars and conferences.

†European University Institute

1 Introduction

In digital marketplaces, established firms use their market power to collect extensive data on their consumers. By using this information to infer consumers' willingness to pay, these firms can tailor prices, potentially diverting demand away from weaker competitors that lack the same access to data. Such practices raise questions about consumer welfare, as they may lead consumers to pay different prices for the same product.[1] Motivated by the welfare impact of such one-sided price personalization, this paper studies how a firm optimally designs an unrestricted information structure to price discriminate among consumers when it competes with a firm that has no access to consumer data.

In 2022, the European Parliament's Internal Market and Consumer Protection committee published a study on personalized pricing, arguing that the practice "can increase competition as it allows traders to provide a competitive offer to consumers, increasing competition between traders. In general, then, increased competition has a positive effect on consumer welfare."[2] This claim draws on studies of personalized pricing in models where one or more firms have perfect information about consumer preferences. A firm that knows each consumer's valuation can attract consumers who prefer a competitor's product by offering targeted price reductions. This aggressive pricing, in turn, forces uninformed competitors to lower their own prices to remain competitive. The resulting downward pressure on prices suggests that using consumer data for price discrimination may benefit consumers.

I show that this argument breaks down once the informed firm chooses how to use its data. When it can freely design its information structure, it opts for a coarser signal than perfect information in order to soften competition. The optimal design deliberately prevents the informed firm from distinguishing consumers who prefer its product from those who prefer its competitor's, thereby limiting its ability to target its competitor's consumers.

[1] While price discrimination may not be explicit nowadays, it occurs in more subtle ways through price discounts or dynamic pricing. See *Does Anyone Know What Paper Towels Should Cost?*, New York Times (February 2022). https://www.nytimes.com/2022/02/26/technology/amazon-price-swings-shopping.html

[2] https://www.europarl.europa.eu/thinktank/en/document/IPOL_STU(2022)734008

Anticipating weaker competitive pressure, the uninformed firm sets a higher price, which in turn allows the informed firm to extract more surplus from its own consumers. Under a monotone loyalty condition, every consumer is worse off than both when neither firm has access to consumer data and when the informed firm perfectly identifies every consumer.

This distinction matters for antitrust authorities. The conventional concern with one-sided price personalization is exclusionary: a dominant platform leverages its data advantage to undercut its competitors and drive them out of the market. In my model, the competitor is never driven out because it can always secure a minimum profit by selling to consumers who prefer its product most. When the informed firm perfectly identifies every consumer, the competitor earns exactly this profit by charging a low price to all its loyal consumers, benefiting aggregate consumer welfare. Under the optimal segmentation, the competitor earns the same profit, but by charging a high price to fewer consumers, which is detrimental to consumers. Indicators based on harm to the competitor, such as its profit or its viability, therefore cannot distinguish the outcome that benefits consumers from the one that harms them. The harm is exploitative because it falls entirely on consumers and arises from a unilateral choice of information structure, without any agreement between firms or any attempt to exclude the competitor.

To demonstrate this effect, I study a model in which two firms compete *à la* Bertrand to sell differentiated goods to a mass of consumers with unit demand, who are privately informed about their valuations for each good. One firm commits to a market segmentation, which allows it to offer a different price in each segment. Its competitor does not observe individual segments, but knows the segmentation and posts a single price, to which the informed firm responds.

I first show that the optimal segmentation can be fully characterized when consumers who prefer the uninformed firm's product more strongly also value it more highly, a condition I call *monotone loyalty* that holds in several workhorse models of differentiation. Under the optimal segmentation, every consumer buys one of the two products (the segmentation is

covered), the uninformed firm serves the consumers who most prefer its product (*U-sorted*), and every remaining consumer buys from the informed firm at their full willingness to pay (*I-extractive*). The uninformed firm posts a higher price than in both the no-segmentation and perfect segmentation cases (Theorem 1). As a result, the informed firm is the only beneficiary of its data: the uninformed firm earns only its profit lower bound, and every consumer is individually weakly worse off (Corollary 1) than in both benchmarks.

This individual-level result contrasts with the existing literature, in which personalized pricing typically creates winners and losers among consumers. For instance, consumers with weak product preferences benefit from lower prices under competitive personalized pricing, whereas consumers with strong preferences pay more (Rhodes and Zhou, 2024). Under monotone loyalty, the optimal segmentation benefits no consumers, as each obtains exactly what the uninformed firm's offer is worth to him, and the offer becomes more expensive.

The optimal segmentation achieves this by pooling consumers who strongly prefer the uninformed firm's product with consumers who have a high willingness to pay for the informed firm's product. In every segment, the informed firm then prefers to charge a high price to its own consumers rather than undercut its competitor to attract the pooled consumers. Anticipating that it will not be undercut, the uninformed firm can safely post a high price and sell to the consumers who prefer its product.

Without monotone loyalty, the market can contain *poor loyalists*: consumers who strongly prefer the uninformed firm's product, but value both products too little to afford it at a high price. The optimal segmentation then no longer has a common structure across markets, but its effect on consumers can still be evaluated. Theorem 2 shows that the optimal segmentation induces a weakly higher price and a weakly lower aggregate consumer surplus than any segmentation in which the uninformed firm serves the consumers who most prefer its product. Since perfect segmentation has this property, consumers are always worse off in aggregate than under perfect segmentation (Corollary 2). Consumers are also worse off than no-segmentation whenever it has this property, but more importantly, whenever every

consumer buys a product under no-segmentation (Corollary 3). Consumers can, however, gain when uniform pricing leaves poor loyalists priced out of both products. In this case, the optimal segmentation induces the uninformed firm to serve these consumers at a lower price, which raises aggregate consumer surplus.

Corollary 3 is interesting because no-segmentation leads every consumer to buy a product when consumers' preferences are relatively homogeneous. These are also the markets in which full information would intensify competition the most: when consumers' relative preferences are close, the informed firm could poach almost any of its competitor's consumers with a small price cut, which would force its competitor to price low. This is precisely why the informed firm refrains from using all its information. The markets in which consumer data would be most pro-competitive are thus those in which its strategic use harms consumers.

The rest of the paper is organized as follows. Section 2 presents an illustrative example that conveys the mechanism behind the optimal segmentation and its welfare consequences. Section 3 introduces the model and the equilibrium concept. Section 4 characterizes the optimal segmentation under monotone loyalty and derives its welfare consequences. Section 5 relaxes monotone loyalty: it compares the optimal segmentation with any U-sorted segmentation, thereby enabling some comparisons with perfect segmentation and no-segmentation, but also presents an example in which consumers gain relative to no-segmentation. Section 6 concludes. The Appendix contains the proofs and an application to the Hotelling line.

1.1 Literature Review

This analysis contributes to the literature on price discrimination in oligopolies. Thisse and Vives (1988), Fudenberg and Tirole (2000), Villas-Boas (1999), and Zhang (2011) study discriminatory pricing strategies in competitive settings and find that personalized prices intensify competition and thereby improve consumer welfare. While these papers assume that firms are symmetrically informed about consumer preferences, others study one-sided information. Shaffer and Zhang (2002) show that one-to-one promotions intensify price

competition, and that a firm that adopts them unilaterally is generally hurt by its rival's response. Liu and Serfes (2004) show that the quality of firms' information about consumer locations has a non-monotonic effect on profits. Belleflamme et al. (2020) find that consumers are better off when only one firm can identify their valuations, even imperfectly, and Braulin (2023) establishes that consumer welfare increases when one firm has perfect information about consumers' bivariate preferences. In all these papers, the information structure is exogenous: firms either perfectly identify consumers or hold a given imperfect information structure. My paper endogenizes the informed firm's information structure. I show that a dominant firm optimally does not use all its information: it coarsens its segmentation to raise its competitor's price, rather than lower it, and extracts all surplus from its own consumers. Under a monotone loyalty condition, this leaves every consumer worse off than under both no-segmentation and perfect segmentation. In the absence of this condition, consumers remain worse off in aggregate than under perfect segmentation, and than under no-segmentation whenever the latter is covered.

Other papers have also observed decreased competition and worsened consumer welfare in the presence of price discrimination. Chen et al. (2001) observe how (exogenously driven) imperfect information about consumers can limit competitive pressure and actually increase a firm's profits. Choe et al. (2024) show that a perfectly informed firm can reduce competition by sharing data with its competitor. Closest to my paper, Rhodes and Zhou (2024) show that consumers can be worse off when only one firm can personalize prices than when either all firms or none can do so, especially when market coverage is high. In their mixed regime, the informed firm personalizes perfectly, and its ability to poach lowers its competitor's uniform price. Their comparisons with uniform pricing hold for specific distributions of preferences. I show that once the informed firm designs its data use, consumers lose whenever the market is covered under uniform pricing, for any distribution of preferences, and that this loss arises because the informed firm raises its competitor's price.

Bergemann et al. (2024), Elliott et al. (2026), and Chen et al. (2026) study how a designer

publicly or privately discloses consumer information to competing firms to maximize their profits. They focus on an intermediary that distributes information to all firms in a market, thereby resulting in an efficient outcome in which consumers buy their preferred products and the entire consumer surplus is extracted. Although the negative impact of strategic information disclosure across multiple firms on consumer welfare is well established, little is known about the consequences of a single firm designing its own supplementary information. As shown in this paper, the effect is also detrimental to consumers, but a single informed firm, by serving some consumers who prefer its competitor's product, attains higher profits at the cost of lower total surplus.

Another strand of literature to which this project contributes is the economics of data. Bergemann et al. (2018) analyze the optimal sale of data to a monopolist. Braulin and Valletti (2016), Bounie et al. (2020), Delbono et al. (2021), and Montes et al. (2019) study how a data broker optimally sells consumer data to competing firms. Closer to my analysis, Bounie et al. (2021) show that a data broker strategically sells partial information on consumers to soften competition between firms. In all these papers, whenever a single firm has access to consumer data, it uses this information to target its competitor's consumers, thereby intensifying competition and raising aggregate consumer surplus relative to the case without information, whether or not this situation arises in equilibrium. These papers, however, restrict the information structures to full information or to segments of a Hotelling line. I show that once the informed firm designs its own information structure without restriction, the opposite holds: it coarsens its information to soften competition, and consumers lose relative to the case without information whenever this case is covered.

Tarantino et al. (Forthcoming) consider a data broker that can sell unrestricted information structures. The data broker's optimal strategy is either to sell fully informative data policies when it is independent, or to keep its advantage of full information when it is integrated into the market. They analyze how a perfectly informed data broker optimally shares data with its competitor, whereas I study how this informed firm optimally manages its

informational advantage ex-ante. I observe that the firm actually prefers not to be perfectly informed to attain an equilibrium in which its profit is higher, while consumer surplus and social welfare are even lower under monotone loyalty.

Finally, my analysis is also linked to the literature on Bayesian persuasion and information design. Building on the seminal work of Kamenica and Gentzkow (2011), Bergemann et al. (2015) characterize the welfare outcomes attainable when a monopolist segments a market, and Haghpanah and Siegel (2023) extend the analysis to multiproduct markets. In competitive settings, Armstrong and Zhou (2022) study how consumers' information about their own valuations shapes the limits of competition, while Elliott et al. (2026) characterize the outcomes an intermediary can induce by informing all firms. I construct the optimal segmentation using the greedy algorithm of Elliott et al. (2026), which builds on Bergemann et al. (2015).

2 An illustrative example

Consider two firms, an informed and an uninformed firm, each selling an indivisible good to a unit mass of consumers with unit demand. Both firms share the common knowledge that consumers' private valuations for the informed and uninformed firm's product are uniformly distributed across two possible pairs: (10, 8), and (8, 10). Consumers' utility from buying a product is their valuation of the product minus the price. The informed firm can split consumers into a finite number of segments[3] and post individual prices in each segment. The uninformed firm, while aware of the segmentation, does not observe the individual segments and therefore posts a single uniform price. The informed firm sets its prices after observing the uninformed firm's price, and indifferent consumers buy the informed firm's product, unless its price is zero. I compare three market segmentation strategies.

[3]As discussed in Section 3, committing to a market segmentation is equivalent to committing to an information structure.

No-Segmentation *(No information).* The informed firm pools all consumers into a single segment and therefore has no informational advantage. The pricing game yields an equilibrium in which the uninformed firm posts a price of 6, to which the informed firm responds with a single price of 8. Figure 1 depicts the market segmentation in panel (a) and the resulting equilibrium in panel (b), where each entry gives the price paid by that type of consumer, and a blue (red) cell indicates that the consumer buys the informed (uninformed) firm's product. At equilibrium, the informed firm sells to consumers of valuation $(10, 8)$, while the uninformed firm sells to consumers of valuation $(8, 10)$.

To see that this is indeed an equilibrium, note that the informed firm is indifferent between posting a price of 8 and posting a price of 4 to poach the entire segment. The uninformed firm is thus undercut whenever it deviates to a higher price. Moreover, the informed firm matches the price decrease if the uninformed firm deviates to a lower price, thereby making the deviation unprofitable as well. Resulting profits are $\Pi_I = 4$ and $\Pi_U = 3$, with consumer welfare $CS = 3$.

	$(10, 8)$	$(8, 10)$
Segment 1	$1/2$	$1/2$

(a) Market segmentation

	$(10, 8)$	$(8, 10)$
Price paid	8	6

(b) Pricing game equilibrium outcome

Figure 1: No-segmentation

Perfect Segmentation *(Full information).* The informed firm divides consumers into two distinct segments based on their valuations, allowing it to perfectly identify each consumer group. As a result, it can undercut the uninformed firm's price when facing consumers with valuation (8, 10). In response, the uninformed firm lowers its price to 2, prompting the informed firm to set a price of 0 in Segment 2. When facing consumers with valuations (10, 8), it posts a price of 4. This reflects a classic result in the literature on price discrimination: to counter the informed firm's ability to poach consumers, the uninformed firm must lower its price to remain competitive. Figure 2 depicts the segmentation and the equilibrium of

the pricing game. In this example, both firms' profits decrease to $\Pi_I = 2$ and $\Pi_U = 1$, while consumers' aggregate welfare increases to $CS = 7$.

	$(10, 8)$	$(8, 10)$
Segment 1	$1/2$	0
Segment 2	0	$1/2$

(a) Market segmentation

	(10, 8)	(8, 10)
Price paid	4	2

(b) Pricing game equilibrium outcome

Figure 2: Perfect Segmentation

Optimal segmentation. The informed firm pools type $(10, 8)$ consumers with mass $1/10$ of type $(8, 10)$ consumers. Remaining consumers are placed in a second segment. By doing so, it deliberately limits its ability to perfectly identify, and thus perfectly price discriminate, some consumers with valuation $(8, 10)$. Anticipating this, the uninformed firm can safely post a price of 10, to which the informed firm responds with a price of 10 in Segment 1. Additionally, the informed firm distinguishes and therefore sells to the remaining type $(8, 10)$ consumers in Segment 2 at a price of 8. Figure 3 depicts the segmentation and the equilibrium of the pricing game.

	$(10, 8)$	$(8, 10)$
Segment 1	$1/2$	$1/10$
Segment 2	0	$4/10$

(a) Market segmentation

	$(10, 8)$	$(8, 10)$	
Price paid	10	8	10

(b) Pricing game equilibrium outcome

Figure 3: Optimal segmentation

To see that this is an equilibrium, consider a deviation by the uninformed firm to a price lower than 10. The informed firm matches the price decrease, if necessary, and keeps all its consumers, so the uninformed firm still sells only to the type $(8, 10)$ consumers of Segment 1, and earns a lower profit. At a price of 2, type $(8, 10)$ consumers now all buy from the uninformed firm, which earns $2 \times 1/2 = 1$, exactly as much as at a price of 10. Any price below 10 therefore earns (weakly) less, and any price above 10 sells nothing.

Relative to the two previous benchmarks, this segmentation strategy raises the informed firm's profit to $\Pi_I = 8.2$, while keeping the uninformed firm's profit at the perfect segmentation level and reducing aggregate consumer surplus to $CS = 0$. Additionally, total surplus falls from 10 to 9.2 because a mass 4/10 of type $(8, 10)$ consumers buy their less-preferred product. This example highlights the limitations of focusing solely on the comparison between no-information and full information regimes: a strategic firm may well prefer a coarser information structure, one that boosts its own profit while reducing consumer surplus. The generalized model introduced in the next section enables to identify markets in which this observation holds surely.

3 Model

Setup. An informed (I) and uninformed (U) firm each produce one differentiated good that they sell to a unit mass of consumers with unit demand. Production costs are assumed to be zero. A consumer's valuation for each good is represented by his private type $\theta = (\theta_I, \theta_U)$ where $\theta_{i\in\{I,U\}}$ takes a value from a finite set $V = \{v_1, ..., v_K\} \subset \mathbb{R}_{++}$ with $v_K < \infty$. A consumer buying the good from firm i at price p_i obtains utility $\theta_i - p_i$. Denote $\Theta = V^2$ the set of all possible types and $\mu \in \Delta(\Theta)$ the *aggregate market* (henceforth the *market*) specifying the initial distribution of consumer types, which is common knowledge.

Market segmentation. Before firms post their prices, the informed firm has the ability to segment the aggregate market into a finite number of *segments*, indexed by $s \in S$ where S is a sufficiently large set. Adopting the notation of Augias et al. (2024), I define a market segmentation as a probability distribution $\sigma \in \Delta(\Theta \times S)$, where $\sigma(\theta, s)$ is the mass of buyers with type θ in segment s. A market segmentation has to satisfy the market consistency

constraint

$$\forall \theta \in \Theta, \quad \sum_{s \in S} \sigma(\theta, s) = \mu(\theta) \tag{MC}$$

Denote $\Sigma(\mu)$ the set of all segmentations satisfying (MC) for the market μ. While the informed firm distinguishes each segment, the uninformed firm knows only μ and σ (but does not observe s). In practice, the informed firm can post a specific price for every segment $s \in S$, whereas the uninformed firm posts only one price.[4] Two benchmark segmentations are (i) *no-segmentation* σ^N where $S = \{s_0\}$ and $\sigma^N(\theta, s_0) = \mu(\theta)$ for all $\theta \in \Theta$, and (ii) *perfect segmentation* σ^P where $|S| = |\Theta|$, and $\sigma^P(\theta, s_\theta) = \mu(\theta)$ and $\sigma^P(\theta', s_\theta) = 0$ for all $\theta' \neq \theta$. The former corresponds to the informed firm having no additional information, while the latter stands for full information.

Timing. The timing of the game is the following

1. The informed firm commits to a publicly observed market segmentation.
2. The uninformed firm posts a unique price p_U.
3. The informed firm observes the price p_U, segment s and posts price $p_I(s, p_U)$.
4. Consumer buys the product that maximizes his utility (or buys nothing).

3.1 Discussion of the assumptions

The model makes two important assumptions that I discuss below.

[4]The market segmentation σ defined above can be interpreted as a signal structure in the sense of Bayesian persuasion, where the message observed by the informed firm is the market index $s \in S$. Formally, $\sigma(\theta, s)$ specifies the probability that a type θ is assigned to segment s. The market-consistency constraint (MC) ensures that σ is an implementable garbling of the underlying type distribution μ. Thus, choosing a segmentation σ is equivalent to choosing a signal structure $\pi(s|\theta) = \sigma(\theta, s)/\mu(\theta)$ mapping types to signals. This representation allows the informed firm to determine how much information about consumers' types to reveal through segmentation, exactly as in an optimal signal design problem.

Commitment to information structure. The commitment assumption seems coherent in situations in which an informed firm, such as Amazon, uses an algorithm to analyze its data and determine the optimal pricing strategy. Pricing algorithms are designed in advance and are not easily revised, which gives the informed firm a form of commitment to how it uses its data. Commitment can also arise from reputation in repeated interactions: an uninformed firm that observes lower profits than expected can infer that the informed firm changed its segmentation and respond by lowering its price, thereby deterring such changes (Mathevet et al., 2026).

Sequential timing. The informed firm sets its prices after observing its competitor's price. This timing is standard in the literature on price discrimination because firms that personalize prices typically rely on more sophisticated pricing technology and can adjust their prices faster than their competitors. Amazon, for instance, uses automated algorithms that continuously adjust prices. Brown and MacKay (2023) document that, in online retail, firms with faster pricing technology systematically respond to price changes by slower rivals. The sequential timing also has a technical advantage. When one firm price discriminates, and its competitor sets a uniform price, simultaneous pricing generally admits no pure-strategy equilibrium, because the uniform-pricing firm's demand jumps discontinuously when it slightly undercuts the personalized offers.

3.2 Equilibrium

Consumers. A consumer buys firm i's product if (i) p_i is below the valuation θ_i, and (ii) the utility of buying from firm i is greater than buying from firm j, i.e. $\theta_i - p_i \geq \theta_j - p_j$. Denote a buyer's willingness to pay (WTP) for product i as

$$w_i(\theta, p_j) = \min\{\theta_i, \theta_i - \theta_j + p_j\}$$

The WTP describes the maximum amount of money a consumer of type θ is willing to pay for product i when competitor j posts price p_j. Notice that the WTP for product i depends on preferences θ, but also on the competitor's price p_j, as a consumer is willing to pay less for a product if the competitor posts a lower price. For the remainder of the paper, the key metric is consumers' WTP for the *informed* firm's product, $w_I(\theta, p_U)$, which I refer to as consumers' WTP hereafter.

I assume that if a consumer is indifferent, he buys the informed firm's product, except when $p_I = 0$. This can be justified by the timing: the informed firm can always undercut its competitor with an infinitesimal lower price to poach an indifferent consumer, except when its price is zero, in which case it cannot undercut anymore. A consumer's decision to buy product i is therefore described by the indicator function

$$B_I(\theta, p_I, p_U) = \begin{cases} 1 & \text{if } w_I(\theta, p_U) > p_I \text{ or } (w_I(\theta, p_U) = p_I > 0) \\ 0 & \text{otherwise} \end{cases}$$

$$B_U(\theta, p_I, p_U) = \big(1 - B_I(\theta, p_I, p_U)\big) \mathbb{1}_{\{\theta_U \geq p_U\}}$$

Firms. The informed firm posts a price in each segment that maximizes its profit given its competitor's price. The informed firm's optimal pricing strategy is given by

$$p_I^*(s, p_U) \in \arg\max_{p_I} \; p_I \sum_{\theta \in \Theta} \sigma(\theta, s) \; B_I(\theta, p_I, p_U) \tag{PM_I}$$

On the other hand, the uninformed firm posts a unique price that maximizes its profit on all the segments, given the informed firm's segmentation and pricing strategy

$$p_U^* \in \arg\max_{p_U} \; p_U \sum_{s \in S} \sum_{\theta \in \Theta} \sigma(\theta, s) \; B_U\big(\theta, p_I^*(s, p_U), p_U\big) \tag{PM_U}$$

Equilibrium. For every segmentation $\sigma \in \Sigma(\mu)$, a *pricing equilibrium* is a pair $\big(p_I(\cdot, \cdot), p_U\big)$ such that $p_I(s, p'_U)$ satisfies (PM_I) for every segment $s \in S$ and every price p'_U, and p_U

satisfies (PM_U) given p_I. Among all pricing equilibria under σ, let (p_I^σ, p_U^σ) denote the informed firm's preferred equilibrium, that is, the one that gives the informed firm the highest profit. Writing $p_I^\sigma(s) := p_I^\sigma(s, p_U^\sigma)$ for the equilibrium price in segment s, let

$$\Pi_I(\sigma) = \sum_{s \in S} p_I^\sigma(s) \sum_{\theta \in \Theta} \sigma(\theta, s)\, B_I\big(\theta, p_I^\sigma(s), p_U^\sigma\big)$$

denote the informed firm's resulting profit under its preferred equilibrium induced by σ. A segmentation σ^* is *optimal* if

$$\sigma^* \in \arg\max_{\sigma \in \Sigma(\mu)} \Pi_I(\sigma)$$

4 Optimal segmentation under Monotone Loyalty

This section studies equilibrium in markets with monotone loyalty: consumers with a stronger preference for the uninformed firm's product also value it more. Under this restriction, the optimal segmentation can be fully characterized, and the equilibrium it induces takes a simple form. The uninformed firm sells only to the consumers who prefer it most, to a mass large enough to earn the same profit as under perfect segmentation. The informed firm serves every remaining consumer and charges each of them their full WTP. The uninformed firm's price is weakly higher than it would be under no-segmentation and perfect segmentation. As a result, the informed firm's profit rises, while the uninformed firm's profit and every consumer's surplus fall weakly relative to both benchmarks.

4.1 Monotone loyal buyers

Under monotone loyal buyers, consumers who prefer the uninformed firm's product more strongly value its product at least as much, as explained in the following Condition. I denote the difference in valuations $\Delta(\theta) = \theta_U - \theta_I$.

Condition 1. (Monotone Loyalty) If $\Delta(\theta') > \max\{\Delta(\theta), 0\}$, then $\theta'_U \geq \theta_U$ for all $\theta, \theta' \in \Theta$.

It is satisfied in several workhorse models of product differentiation.

Hotelling Line. In Thisse and Vives (1988), consumers' valuations are anti-correlated with $\theta_I = V - \tau(x)$ and $\theta_U = V - \tau(1-x)$ for a parameter $V > 0$ and increasing τ. Consumers closer to U have a higher $\Delta(\theta)$ and θ_U. More generally, any pure horizontal model with a constant $\theta_I + \theta_U$ satisfies the Monotone Loyalty condition as $\theta_U = \frac{\theta_U + \theta_I + \Delta(\theta)}{2}$ is increasing in $\Delta(\theta)$.

Pure vertical differentiation. In Shaked and Sutton (1982), consumers have types $\theta_i = q_i s$ where s is a random taste-for-quality parameter. If $q_U > q_I$, then $\Delta(\theta) = (q_U - q_I)s$ and $\theta_U = q_U s$ are both increasing in s. If $q_U < q_I$, every consumer has $\Delta(\theta) < 0$, and the condition holds trivially.

Consumers with switching costs. In Klemperer (1987), both firms sell an identical good that all consumers value at a common valuation v, but each consumer is attached to one firm and incurs a switching cost $c \geq 0$, which may differ across consumers, when buying from the other firm. A consumer attached to the informed firm has type $(v, v-c)$, so $\Delta(\theta) = -c \leq 0$, and a consumer attached to the uninformed firm has type $(v-c, v)$, so $\Delta(\theta) = c$. Every consumer with $\Delta(\theta) > 0$ is therefore attached to the uninformed firm and values its product at v, the highest valuation in the market, so Condition 1 holds.

Common valuation for one product. If $\theta_U = \overline{v}$ for all θ, then $\theta'_U \geq \theta_U$ trivially. If $\theta_I = \overline{v}$, then $\Delta(\theta) = \theta_U - \overline{v}$ and a higher $\Delta(\theta)$ means a higher θ_U.

Richer consumers are more loyal to U. Suppose that every consumer who prefers the uninformed firm's product has a valuation $\theta_U = a + \phi\big(\Delta(\theta)\big)$, for a constant a and a function ϕ that is nondecreasing on $[0, \infty)$, and that every other consumer has $\theta_U \leq a + \phi(0)$.

If $\Delta(\theta') > \max\{\Delta(\theta), 0\}$, then $\theta'_U = a + \phi\big(\Delta(\theta')\big)$. This is at least $a + \phi\big(\Delta(\theta)\big) = \theta_U$ if $\Delta(\theta) > 0$, and at least $a + \phi(0) \geq \theta_U$ otherwise, so Condition 1 holds. Consumers who prefer the uninformed firm's product more strongly thus value it more highly, while consumers who prefer the informed firm's product are unrestricted, as long as they do not value the uninformed firm's product more than its least loyal customers.

Condition 1 fails when relative preferences and valuation vary independently. This is the case, for instance, in random-utility models in which valuations are drawn independently for each product (Perloff and Salop, 1985): a consumer may strongly prefer the uninformed firm's product while valuing both products little.

4.2 Characteristics of optimal segmentation

The characterization of the optimal segmentation under Condition 1 relies on the following three properties. Each refers to the equilibrium prices $p_I^\sigma(s)$ and p_U^σ induced by a segmentation σ, and I write $B_I^\sigma(\theta, s)$ and $B_U^\sigma(\theta, s)$ for the purchase decisions of a consumer of type θ in segment s at these prices.

Definition 4.1. A segmentation σ is *covered* if every consumer buys one of the two products, that is, $B_I^\sigma(\theta, s) + B_U^\sigma(\theta, s) = 1$ for every (θ, s) with $\sigma(\theta, s) > 0$.

Equivalently, every consumer who cannot afford the uninformed firm's product, $\theta_U < p_U^\sigma$, buys from the informed firm, $\theta_I \geq p_I^\sigma(s)$.

Definition 4.2. A segmentation σ is *I-extractive* if

$$p_I^\sigma(s) \;\geq\; w_I(\theta, p_U^\sigma) \qquad \text{for every } (\theta, s) \text{ with } \sigma(\theta, s) > 0.$$

In an I-extractive segmentation, every consumer of the informed firm pays exactly his WTP.

Definition 4.3. A segmentation σ is *U-sorted* if there is a threshold $\hat{\delta}$ such that, for every

(θ, s) with $\sigma(\theta, s) > 0$,

$$\Delta(\theta) > \hat{\delta} \implies B_U^\sigma(\theta, s) = 1 \qquad \text{and} \qquad \Delta(\theta) < \hat{\delta} \implies B_U^\sigma(\theta, s) = 0.$$

In a U-sorted segmentation, the uninformed firm serves every consumer whose relative preference for its product exceeds the threshold, serves no consumer below it, and possibly some consumers at the threshold. In other words, it serves the consumers who prefer its product the most.

Theorem 1. Under Condition 1, every optimal segmentation is covered, I-extractive and U-sorted, and induces the uninformed firm to post a weakly higher price than under no-segmentation and under perfect segmentation.

Intuition of the proof. The proof is done in three steps. I first present an important definition and then give an intuition of each step.

Natural turf and price floor. Consumers who strongly prefer the uninformed firm's product can never be poached by the informed firm if p_U is sufficiently low: at that price, their WTP is weakly negative. These consumers form the uninformed firm's *natural turf*. Whatever the segmentation, the uninformed firm can always sell to its natural turf, which secures it a minimum profit $\underline{\Pi_U}$, obtained at price $\underline{p_U}$. Perfect segmentation achieves exactly this lower bound. When every type forms its own segment, the informed firm charges each consumer outside the natural turf their full WTP and serves them, while the uninformed firm sells only to its natural turf and optimally posts $\underline{p_U}$.

Consider now a segmentation inducing a price $p_U < \underline{p_U}$. The uninformed firm must still earn at least $\underline{\Pi_U}$, so it sells to a mass of at least $\underline{\Pi_U}/p_U$, strictly more than under perfect segmentation. Compared with perfect segmentation, the informed firm therefore (i) loses consumers who would have paid it a positive price, and (ii) faces a lower competing price, which lowers the WTP of every consumer it keeps. Both effects reduce its profit. Hence,

perfect segmentation dominates every segmentation inducing a price below $\underline{p_U}$, and the price under the optimal segmentation cannot fall below it.

Step 1: U-sortedness. At a given price p_U, the cheapest consumers for the informed firm to cede to its competitor are those with the lowest WTP among the consumers who can afford the uninformed firm's product, that is, those with the highest $\Delta(\theta)$ among them. Condition 1 guarantees that these are also the consumers with the highest $\Delta(\theta)$ in the whole market.

Step 2: The optimal segmentation is covered, I-extractive and U-sorted. Fix a price $p_U \geq \underline{p_U}$. The informed firm's profit is maximized at that price when three conditions hold. First, the uninformed firm receives exactly the mass $\underline{\Pi_U}/p_U$ it needs to earn its lower bound. Second, this mass consists of consumers with the lowest WTP, that is, those with the largest difference in valuation $\Delta(\theta)$. Third, the informed firm serves everyone else at their full WTP. A segmentation that is not U-sorted, not I-extractive, or not covered violates these conditions. The question is whether a segmentation can attain this bound in equilibrium. Two requirements must hold: the informed firm must prefer to surrender the uninformed firm's consumers rather than poach them, and the uninformed firm must not want to lower its price.

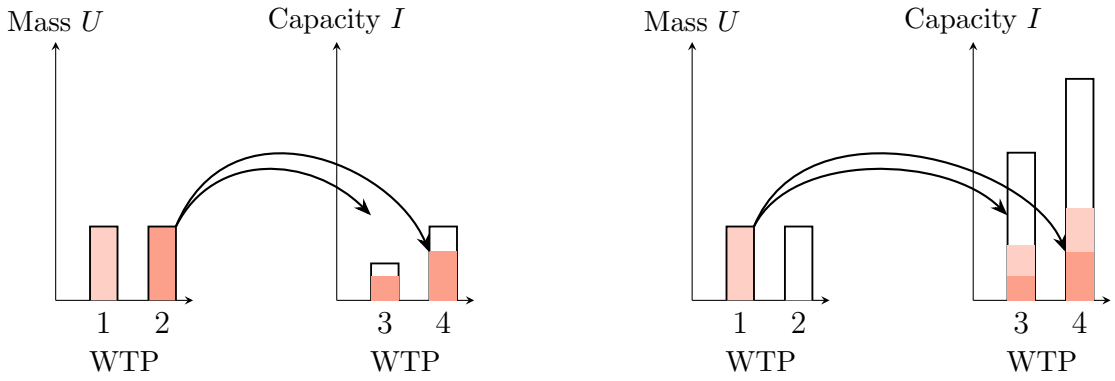
Poaching is deterred by pooling. Consider a segment that contains a mass a of the informed firm's consumers with WTP y, and a mass b of the uninformed firm's consumers with a lower WTP $z < y$. If the informed firm charges y, it sells only to its own consumers and earns ya. If it cuts its price to z, it also poaches the uninformed firm's consumers and earns $z(a+b)$. It therefore keeps the high price as long as $ya \geq z(a+b)$, or

$$b \;\leq\; a\,\frac{y-z}{z}$$

Following Elliott et al. (2026), I call the right-hand side the *capacity* of consumers with WTP

y at level z: it is the largest mass of the uninformed firm's consumers with WTP at least z that can be pooled with them without triggering poaching.

The segmentation algorithm fills capacities from the top, as illustrated in Figure 4. Suppose WTPs equal $1, 2, 3, 4$, each with mass $1/4$, and that at price p_U the uninformed firm must serve the consumers with WTP 1 and 2 to earn its profit lower bound. For the informed firm's consumers with WTP $y = 4$ and mass $a = 1/4$, the capacity is $1/4$ at level $z = 2$ and $3/4$ at level $z = 1$. For consumers with WTP $y = 3$, the corresponding capacities are $1/8$ and $1/2$. The algorithm first splits the consumers with WTP 2 in proportion to the available capacities: $1/6$ is pooled with WTP 4 and $1/12$ with WTP 3 (panel (a)). It then splits the consumers with WTP 1 in proportion to the remaining capacities, $3/4 - 1/6 = 7/12$ and $1/2 - 1/12 = 5/12$, which gives $7/48$ and $5/48$ (panel (b)). The algorithm never runs out of capacity. If some segmentation induces price p_U, the informed firm already chooses not to poach there, and this choice implies that total capacity suffices at every WTP level.



(a) Distributing consumers with WTP 2 (b) Distributing consumers with WTP 1

Figure 4: Algorithm to construct optimal segmentation

In the resulting segmentation, the informed firm extracts the full WTP of its consumers and surrenders the required mass to its competitor. Undercutting does not pay for the uninformed firm. A lower p_U reduces the WTP of the uninformed firm's consumers by at least as much as it does for the informed firm's consumers, so poaching is still deterred in

every segment. The uninformed firm keeps its consumers and gains at most its natural turf at the lower price. Because its consumers are those with the highest $\Delta(\theta)$ by Step 1, the two sets are nested, so it earns at most $\underline{\Pi_U}$. The uninformed firm may instead prefer to raise its price, which benefits the informed firm: a higher p_U weakly raises every consumer's WTP, so the informed firm earns at least as much in every segment.

Applied at the optimal price p_U^*, this construction shows that the informed firm can reach its profit upper bound. Every optimal segmentation must therefore attain it, and so must be covered, I-extractive and U-sorted.

Step 3: Price comparison. The price floor already shows that the uninformed firm's price under an optimal segmentation is weakly greater than under perfect segmentation. It also settles the comparison with no-segmentation when $p_U^N < \underline{p_U}$.

Suppose instead that $p_U^N \geq \underline{p_U}$. No-segmentation is itself a segmentation, inducing p_U^N, so there is enough capacity at p_U^N. The algorithm of Step 2, therefore, yields a covered, I-extractive, and U-sorted segmentation that gives the informed firm the highest profit attainable at p_U^N. This highest attainable profit strictly increases with the uninformed firm's price, for two reasons. First, a higher p_U weakly raises every consumer's WTP. Second, the uninformed firm needs to serve a smaller mass, $\underline{\Pi_U}/p_U$, to earn its lower bound, so the informed firm retains more consumers. Any segmentation inducing a price below p_U^N thus earns strictly less than the segmentation constructed at p_U^N, and cannot be optimal. Hence, the optimal segmentation induces a price at least as high as under no-segmentation. □

Theorem 1 reduces the design of the optimal segmentation to a one-dimensional problem. Since the highest profit the informed firm can attain at a given price strictly increases with the uninformed firm's price, the optimal segmentation induces the highest price $p_U \geq \underline{p_U}$ at which the informed firm can cede its cheapest consumers to its competitor in equilibrium. This is the highest price at which enough consumers can afford the uninformed firm's product for it to earn its profit lower bound, and the informed firm's consumers have enough capacity

to absorb those it surrenders. Two forces therefore cap the uninformed firm's price: either too few consumers can afford its product for it to earn its profit lower bound, or the informed firm's consumers no longer have enough capacity to hide those it must surrender.

The theorem also shows how the informed firm uses its information. It never poaches its competitor's consumers in equilibrium. Instead, it uses its information to hide them among its own consumers, which commits it not to poach them and allows its competitor to raise its price. The informed firm then serves every consumer whose relative preference for its competitor's product falls below the threshold, including some consumers who prefer the competitor's product. The optimal segmentation, therefore, misallocates these consumers, which lowers total surplus, while the competitor's profit remains at its lower bound.

4.3 Welfare outcome

The welfare consequences of segmentation follow from two features of Theorem 1: the price increase and the I-extractive property of the optimal segmentation.

Corollary 1. Under Condition 1, compared with both no-segmentation and perfect segmentation, the optimal segmentation weakly increases the informed firm's profit, weakly decreases the uninformed firm's profit, and weakly decreases the surplus of every consumer.

Proof. The informed firm's profit is weakly higher because both benchmarks are feasible segmentations. The uninformed firm earns at least its profit lower bound $\underline{\Pi_U}$ under any segmentation, by Lemma 1, and exactly $\underline{\Pi_U}$ under the optimal segmentation, by Step 2 of the proof of Theorem 1.

Finally, under the optimal segmentation, a consumer buying from the informed firm obtains $\theta_I - w_I(\theta, p_U^*) = \max\{\theta_U - p_U^*, 0\}$, and a consumer buying from the uninformed firm obtains $\theta_U - p_U^* \geq 0$. Under any segmentation in which the uninformed firm posts p_U, a consumer can buy from it or buy nothing, so he obtains at least $\max\{\theta_U - p_U, 0\}$. Since $p_U^* \geq p_U^N$ and $p_U^* \geq \underline{p_U}$ by Theorem 1, every consumer is weakly worse off than under both benchmarks. □

Corollary 1 is stronger than an aggregate welfare comparison. Price discrimination typically creates winners and losers among consumers, and its welfare effect depends on how gains and losses balance. Under Condition 1, the optimal segmentation creates no benefits to consumers: every consumer obtains exactly what the uninformed firm's offer is worth to him, $\max\{\theta_U - p_U^*, 0\}$, and this offer becomes more expensive. Consumer surplus is therefore determined by a single price, that of the uninformed firm, and the informed firm's data harms consumers only through its effect on that price. This channel differs from the one at work when the informed firm perfectly personalizes prices, as in the mixed regime of Rhodes and Zhou (2024): there, the informed firm's ability to poach lowers its competitor's price, which benefits the consumers who prefer the competitor's product. Here, the informed firm uses its information to raise its competitor's price, and every consumer is worse off, whether or not the market is covered. In the next section, I relax Condition 1 and show that consumers remain worse off in aggregate than under perfect segmentation for any type distribution, and than under no-segmentation whenever no-segmentation is U-sorted.

5 Beyond Monotone Loyalty

Without Condition 1, the market can contain *poor loyalists*: consumers who strongly prefer the uninformed firm's product but value both products too little to afford it at a high price. The optimal segmentation then no longer has a common structure: it may leave consumers unserved, leave rent to the informed firm's consumers, or fail to be U-sorted. Its effect on consumers can nevertheless still be evaluated in some aggregate markets. I show that aggregate consumer surplus is weakly lower under the optimal segmentation than under any U-sorted segmentation. In particular, consumers always lose relative to perfect segmentation, and lose relative to no-segmentation whenever it is U-sorted (for instance, when it is covered). Consumers can, however, gain when no-segmentation is not U-sorted: when rich consumers pull the informed firm's uniform price up, and poor loyalists are priced

out of both products, the optimal segmentation lowers the uninformed firm's price and increases aggregate consumer surplus.

5.1 Comparison to U-sorted segmentation

The first result compares the optimal segmentation with any segmentation under which the uninformed firm serves the consumers who most prefer its product.

Theorem 2. Every optimal segmentation σ^* induces a weakly higher price for the uninformed firm, and a weakly lower aggregate consumer surplus than any U-sorted segmentation σ.

Intuition of the proof. Consider a U-sorted segmentation σ inducing price p_U, and the following benchmark outcome at this price: the uninformed firm serves exactly the mass $\underline{\Pi_U}/p_U$ of consumers it needs to earn its profit lower bound, chosen among those with the lowest WTP for the informed firm's product, and the informed firm serves every other consumer at his full WTP. Because σ is U-sorted, the algorithm of Theorem 1 builds a segmentation under which the informed firm earns at least the profit of this benchmark: the uninformed firm has no incentive to lower its price, and if it raises it, the informed firm's profit only increases. I write CS^B and Π_I^B for consumer surplus and the informed firm's profit in the benchmark. The optimal segmentation σ^* therefore gives the informed firm at least the profit of the benchmark Π_I^B, and, by the argument of Theorem 1, induces a price $p_U^* \geq p_U$.

Consider the joint surplus of consumers and the informed firm, $CS + \Pi_I$. A consumer buying from the informed firm contributes his valuation θ_I, a consumer buying from the uninformed firm contributes $\theta_U - p_U$, and a consumer buying nothing contributes nothing. Ceding a consumer to the uninformed firm therefore changes this joint surplus by $\Delta(\theta) - p_U$, that is, by minus his WTP. The joint surplus thus equals the total valuation for the informed firm's product, plus the sum of $\Delta(\theta)$ over the uninformed firm's consumers, minus the uninformed firm's profit and the valuation of excluded consumers.

For a given price p_U, the joint surplus is highest when no consumer is excluded, and the uninformed firm is ceded the smallest mass of consumers it needs to earn $\underline{\Pi_U}$, namely $\underline{\Pi_U}/p_U$, chosen among those with the lowest WTP for the informed firm's product. Indeed, consumers with a negative WTP form the natural turf, which the uninformed firm serves anyway, and every other consumer ceded lowers the joint surplus. The benchmark outcome attains this maximum at p_U. Consider now an outcome with the same properties at a higher price $p'_U > p_U$. Since the uninformed firm earns the same profit in both outcomes, the difference in joint surplus is determined by the sum of $\Delta(\theta)$ over the uninformed firm's consumers. At p'_U, the uninformed firm serves fewer consumers, and only among those who can afford its product at the higher price. At p_U, it can serve all of them, and the additional consumers it serves prefer its product. The sum of $\Delta(\theta)$ over the ceded consumers is therefore weakly larger at p_U, and so is the joint surplus. The maximum joint surplus thus decreases weakly with the uninformed firm's price. Since the joint surplus under σ^* is at most the maximum at $p_U^* \geq p_U$, it is weakly smaller than in the benchmark. Moreover, since the informed firm earns a weakly higher profit under σ^* than in the benchmark,

$$CS(\sigma^*) + \Pi_I(\sigma^*) \leq CS^B + \Pi_I^B \quad \Rightarrow \quad CS(\sigma^*) \leq CS^B$$

Finally, the informed firm extracts the full WTP of its consumers in the benchmark, so every consumer obtains exactly what the uninformed firm's offer is worth to him at p_U, namely $\max\{\theta_U - p_U, 0\}$. Under σ, every consumer can still buy from the uninformed firm at p_U or buy nothing, so he obtains at least this amount. Hence $CS^B \leq CS(\sigma)$, and $CS(\sigma^*) \leq CS(\sigma)$. □

Two results follow directly.

Corollary 2. Every optimal segmentation induces a weakly higher price for the uninformed firm and a weakly lower aggregate consumer surplus than perfect segmentation.

Proof. Perfect segmentation is U-sorted: the uninformed firm serves exactly its natural turf

$NT(\underline{p_U}) = \{\theta : \Delta(\theta) \geq \underline{p_U}\}$. □

Corollary 2 echoes Bounie et al. (2021), who show that a data broker selling information to competing firms does not sell full information to a firm, but strategically withholds information on some consumers in order to soften competition, which leaves consumers worse off than when one firm is fully informed. The same logic also applies in this analysis: the informed firm prefers not to use all its information, and consumers are worse off in aggregate than under full personalization. However, unlike in their paper, when every consumer buys under uniform pricing, consumers are also worse off than when neither firm has an informational advantage, as the next result shows.

Corollary 3. If no-segmentation is U-sorted or covered, every optimal segmentation induces a weakly higher price for the uninformed firm and a weakly lower aggregate consumer surplus than no-segmentation.

Proof. The first (U-sorted) statement follows from Theorem 2. For the second (coveredness), I show that no-segmentation is U-sorted whenever it is covered. If $p_I^N = 0$, no consumer has a strictly positive WTP, since the informed firm could otherwise earn a positive profit. Every consumer thus has $\theta_U > \theta_I$ and buys from the uninformed firm, so no-segmentation is U-sorted with threshold p_U^N. Assume therefore $p_I^N > 0$. Let θ buy from the uninformed firm and θ' not, under no-segmentation. Then $\theta_U \geq p_U^N$ and $w_I(\theta, p_U^N) = p_U^N - \Delta(\theta) < p_I^N$, so $\Delta(\theta) > p_U^N - p_I^N$. If $\theta'_U \geq p_U^N$, then θ' buys from the informed firm, so $p_U^N - \Delta(\theta') = w_I(\theta', p_U^N) \geq p_I^N$, that is, $\Delta(\theta') \leq p_U^N - p_I^N$. If $\theta'_U < p_U^N$, coverage gives $\theta'_I \geq p_I^N$, so $\Delta(\theta') < p_U^N - p_I^N$. In both cases $\Delta(\theta') < \Delta(\theta)$, so no-segmentation is U-sorted with threshold $p_U^N - p_I^N$. □

Whether no-segmentation is covered depends on the dispersion of consumers' valuations. When valuations are relatively homogeneous, neither firm has much to gain from pricing out consumers with low valuations: lowering a uniform price to serve them costs little on the consumers it already serves. Each consumer can then afford at least one of the two products, so no-segmentation is covered, and hence U-sorted. Since coverage requires only that each

consumer buy one of the two products, homogeneous valuations for either product already push in this direction. Exclusion requires dispersed valuations for both products: typically, a small segment with a much higher valuation for the informed firm's product pulls up its uniform price, while consumers value the uninformed firm's product too little to afford it. When these excluded consumers are poor loyalists who strongly prefer the uninformed firm's product despite their low valuation, no-segmentation fails to be U-sorted, and consumers may gain from segmentation, as the next subsection will show.

Corollary 3 contrasts with the main lesson of the literature on competitive price discrimination. When all firms can personalize prices, high market coverage is precisely what makes price discrimination pro-competitive: firms compete for each consumer individually, and consumers gain (Thisse and Vives, 1988; Rhodes and Zhou, 2024). When only one firm has access to consumer data and decides how to use it, coverage has the opposite effect: it reduces aggregate consumer welfare. The result requires no assumption about the distribution of preferences and holds regardless of the form the optimal segmentation takes, even though the latter cannot be characterized in general. Finally, the corollary identifies the only channel through which consumers can gain from segmentation: uniform pricing must leave some consumers unserved, and the optimal segmentation must bring them into the market.

5.2 Welfare improvement in the presence of poor loyalists

Consumer welfare may improve under the optimal segmentation when no-segmentation is not U-sorted. Consider the aggregate market presented in Table 1 that contains three different types: consumers who highly value the informed firm's product, consumers who are almost indifferent between both products, and consumers who prefer the uninformed firm's product but do not value it highly, i.e., the poor loyalists.

No-segmentation. When the uninformed firm posts $p_U = 10$, the informed firm prefers to target the rich consumers: charging 1000 yields 100, while charging type $(9.9, 10)$ consumers' WTP yields $9.9 \times 0.96 \approx 9.50$. The uninformed firm then sells to type $(9.9, 10)$ consumers

θ	(1000, 10)	(9.9, 10)	(0.5, 9.5)
$\mu(\theta)$	0.1	0.86	0.04
$\Delta(\theta)$	-990	0.1	9

Table 1: Aggregate market

at $p_U^N = 10$ and earns 8.6. Lowering its price to 9.5 to also attract type $(0.5, 9.5)$ consumers would only yield $9.5 \times 0.90 = 8.55$. The poor loyalists are therefore priced out of both products, and no-segmentation is not U-sorted: type $(0.5, 9.5)$ consumers have the highest $\Delta(\theta)$ but do not buy from the uninformed firm. Every other consumer pays his full valuation, so $CS^N = 0$. The pricing game equilibrium is depicted in Figure 5.

Optimal segmentation. The uninformed firm's profit lower bound $\underline{\Pi_U} = 9 \times 0.04 = 0.36$ is attained at $\underline{p_U} = 9$. The optimal segmentation induces price $p_U^* = 9.5$ and is presented in Table 2.

θ	(1000, 10)	(9.9, 10)	(0.5, 9.5)	$p_I^*(s)$	p_U^*
$\sigma(\theta, s_1)$	0.1	0	0	999.5	9.5
$\sigma(\theta, s_2)$	0	0.86	18/475	9.4	9.5
$\sigma(\theta, s_3)$	0	0	1/475	0.5	9.5

Table 2: Optimal segmentation

In segments s_1 and s_3, the informed firm perfectly distinguishes the consumers and charges their WTP. In segment s_2, posting price $p_I = 9.4$ yields $9.4 \times 0.86 \approx 8.08$, which is more than posting price $p_I = 0.5$ yielding 0.45, so the informed firm surrenders type $(0.5, 9.5)$ consumers to its competitor.

The uninformed firm sells to a mass 18/475 of consumers and earns exactly its profit lower bound $18/475 \times 9.5 = 0.36$. It has no profitable deviation. At a price $p_U \in (9, 9.5)$, the informed firm matches the price decrease and the uninformed firm sells to the same consumer mass at a lower price. Lowering it to 9 turns all type $(0.5, 9.5)$ consumers into its natural turf, which yields exactly 0.36. Any lower price yields $0.04\, p_U < 0.36$, and any higher price sells nothing. The informed firm earns $\Pi_I \approx 108.035$.

No other price does better. Every segmentation that induces a price below 9.5 yields the informed firm a strictly lower profit because it can extract less surplus and must surrender more consumers to its competitor. At any price $p_U \in (9.5, 10]$, type (0.5, 9.5) consumers cannot afford the uninformed firm's product, so the informed firm must surrender a mass N of type (9.9, 10) or (1000, 10) consumers. If the uninformed firm deviates to $p_U = 9$, the WTP of these consumers falls by exactly $p_U - 9$, while the WTP of every consumer the informed firm keeps falls by at most this amount. As in the proof of Lemma 4 in the Appendix, the informed firm therefore still prefers not to poach them, and the uninformed firm keeps them. At $p_U = 9$, type (0.5, 9.5) consumers also become its natural turf, so the uninformed firm earns at least $9(N + 0.04)$ by deviating. It must therefore earn at least as much on path, so $p_U N \geq 9(N + 0.04)$, that is, $N \geq 0.36/(p_U - 9) \geq 0.36$. The informed firm's profit is therefore at most $0.1 \times (990 + p_U) + (p_U - 0.1)(0.86 - 0.36) + 0.5 \times 0.04$, which is increasing in p_U and equals $104.97 < 108.035$ at $p_U = 10$.

The pricing game equilibrium under no-segmentation and the optimal segmentation are depicted in Figure 5. Blue cells indicate consumers buying from the informed firm, red cells indicate consumers buying from the uninformed firm, and white cells indicate consumers buying nothing. Under the optimal segmentation, a share 1/19 of type (0.5, 9.5) buys from the informed firm at price 0.5, and the rest buys from the uninformed firm at price 9.5.

	(1000, 10)	(9.9, 10)	(0.5, 9.5)
Price	1000	10	–

(a) No-segmentation

	(1000, 10)	(9.9, 10)	(0.5, 9.5)
Price	999.5	9.4	9.5

(b) Optimal segmentation

Figure 5: Pricing game equilibrium

Welfare. The optimal segmentation is covered and I-extractive, so every consumer obtains $\max\{\theta_U - 9.5, 0\}$, and

$$CS(\sigma^*) = (0.1 + 0.86) \times 0.5 = 0.48 \ > \ 0 = CS^N$$

Consumers gain because segmentation lowers the uninformed firm's price from 10 to 9.5. Under no-segmentation, consumers with a high valuation for the informed firm's product raise its price, leaving the poor loyalists priced out of both products. Under the optimal segmentation, the cheapest way to secure the uninformed firm's profit lower bound is to cede these poor loyalists at their valuation, which requires a lower price for the uninformed firm and benefits consumers who buy from the informed firm. Consistent with Corollary 2, consumers remain worse off than under perfect segmentation, where $\underline{p_U} = 9$ and $CS^P = 0.96 \times 1 + 0.04 \times 0.5 = 0.98$.

The example also shows that the structure of the optimal segmentation is not what drives the welfare gain: although Condition 1 fails, the optimal segmentation is covered, I-extractive and U-sorted. Consumers gain because no-segmentation is not U-sorted.

6 Concluding remarks

I studied how a firm optimally uses consumer data when it competes against a firm that has no access to such data. A common view holds that personalized pricing intensifies competition and therefore benefits consumers. I showed that a strategic firm instead coarsens its information to induce its competitor to raise its price. The higher price allows the informed firm to retain more consumers, since its competitor needs fewer of them to secure its profit, and to extract more surplus from them. Under a monotone loyalty condition, every consumer is worse off than under both uniform pricing and full personalization, and the competitor earns only the profit it can secure from its most loyal consumers. Without this condition, consumers remain worse off in aggregate than under any segmentation in which the competitor serves the consumers who most prefer its product, which includes full personalization and, in fully covered markets, uniform pricing.

These results bear on how antitrust authorities assess data-rich firms. The harm identified here is exploitative rather than exclusionary: the competitor is not driven out of the

market, and, under monotone loyalty, it earns the same profit as under full personalization. Assessments based on harm to competitors, therefore, cannot distinguish the outcome that benefits consumers from the one that harms them. Only an assessment based on the prices consumers pay can.

This analysis has several limitations. First, the model considers a single informed firm facing a single uninformed competitor. Allowing for several competitors, possibly with different degrees of access to data, could yield richer strategic interactions and affect the welfare implications. Second, the informed firm can commit to any segmentation at no cost. In practice, acquiring and processing data is costly, and the informed firm may not be able to commit to how it uses its data. Modeling these frictions could alter both firms' incentives. Third, consumers play no strategic role: they cannot withhold their data or conceal their preferences, as studied in other settings by Ali et al. (2023). These limitations point to promising directions for future research, in particular endogenous data acquisition, consumers' control over their data, and more complex competitive environments.

References

Ali, S. N., G. Lewis, and S. Vasserman (2023): "Voluntary disclosure and personalized pricing," *The Review of Economic Studies*, 90, 538–571.

Armstrong, M. and J. Zhou (2022): "Consumer Information and the Limits to Competition," *American Economic Review*, 112, 534–77.

Augias, V., D. M. Barreto, and A. Ghersengorin (2024): "Redistribution Through Market Segmentation," in *Proceedings of the 25th ACM Conference on Economics and Computation*, 1185–1186.

Belleflamme, P., W. M. W. Lam, and W. Vergote (2020): "Competitive Imperfect Price Discrimination and Market Power," *Marketing Science*, 39, 996–1015.

Bergemann, D., A. Bonatti, and A. Smolin (2018): "The Design and Price of Information," *American Economic Review*, 108, 1–48.

Bergemann, D., B. Brooks, and S. Morris (2015): "The Limits of Price Discrimination," *American Economic Review*, 105, 921–957.

——— (2024): "On the Alignment of Consumer Surplus and Total Surplus under Competitive Price Discrimination," .

Bounie, D., A. Dubus, and P. Waelbroeck (2020): "Market for Information and Selling Mechanisms," *Available at SSRN 3454193*.

——— (2021): "Selling Strategic Information in Digital Competitive Markets," *The RAND Journal of Economics*, 52, 283–313.

Braulin, F. C. (2023): "The Effects of Personal Information on Competition: Consumer Privacy and Partial Price Discrimination," *International Journal of Industrial Organization*, 87, 102923.

Braulin, F. C. and T. Valletti (2016): "Selling Customer Information to Competing Firms," *Economics Letters*, 149, 10–14.

Brown, Z. Y. and A. MacKay (2023): "Competition in pricing algorithms," *American Economic Journal: Microeconomics*, 15, 109–156.

Chen, Y., C. Narasimhan, and Z. J. Zhang (2001): "Individual Marketing with Imperfect Targetability," *Marketing Science*, 20, 23–41.

Chen, Y., X. Shi, and J. Zhang (2026): "Welfare of Competitive Price Discrimination with Captive Consumers," *American Economic Journal: Microeconomics*.

Choe, C., J. Cong, and C. Wang (2024): "Softening Competition through Unilateral Sharing of Customer Data," *Management Science*.

Delbono, F., C. Reggiani, and L. Sandrini (2021): "Strategic Data Sales to Competing Firms," Tech. rep., JRC Digital Economy Working Paper.

Elliott, M., A. Galeotti, A. Koh, and W. Li (2026): "Market Segmentation through Information," *Review of Economic Studies*.

Fudenberg, D. and J. Tirole (2000): "Customer Poaching and Brand Switching," *RAND Journal of Economics*, 634–657.

Haghpanah, N. and R. Siegel (2023): "Pareto-improving Segmentation of Multiproduct Markets," *Journal of Political Economy*, 131, 1546–1575.

Kamenica, E. and M. Gentzkow (2011): "Bayesian Persuasion," *American Economic Review*, 101, 2590–2615.

Klemperer, P. (1987): "Markets with Consumer Switching Costs," *The Quarterly Journal of Economics*, 102, 375–394.

Liu, Q. and K. Serfes (2004): "Quality of Information and Oligopolistic Price Discrimination," *Journal of Economics & Management Strategy*, 13, 671–702.

Mathevet, L., D. Pearce, and E. Stacchetti (2026): "Reputation and information design," *Journal of Political Economy*.

Montes, R., W. Sand-Zantman, and T. Valletti (2019): "The value of personal information in online markets with endogenous privacy," *Management Science*, 65, 1342–1362.

Perloff, J. M. and S. C. Salop (1985): "Equilibrium with Product Differentiation," *The Review of Economic Studies*, 52, 107–120.

Rhodes, A. and J. Zhou (2024): "Personalized Pricing and Competition," *American Economic Review*, 114, 2141–2170.

Shaffer, G. and Z. J. Zhang (2002): "Competitive One-to-one Promotions," *Management Science*, 48, 1143–1160.

Shaked, A. and J. Sutton (1982): "Relaxing Price Competition through Product Differentiation," *The Review of Economic Studies*, 49, 3–13.

Tarantino, E., S. Terstiege, and A. Vigier (Forthcoming): "Market Segmentation, Information Sale, and Information Foreclosure," *RAND Journal of Economics*.

Thisse, J.-F. and X. Vives (1988): "On the strategic choice of spatial price policy," *The American Economic Review*, 122–137.

Villas-Boas, J. M. (1999): "Dynamic Competition with Customer Recognition," *The Rand Journal of Economics*, 604–631.

Zhang, J. (2011): "The Perils of Behavior-based Personalization," *Marketing Science*, 30, 170–186.

A Appendix

I introduce some definitions used in the proofs of the theorems and corollaries.

Natural turf. For $p_U > 0$, define the uninformed firm's *natural turf* as consumers that the informed firm can never poach when the uninformed firm posts price p_U, and the set of consumers who *can afford* its product as

$$NT(p_U) = \{\theta : \Delta(\theta) \geq p_U\}, \quad \text{and} \quad K(p_U) = \{\theta : \theta_U \geq p_U\}$$

Notice that $NT(p_U) \subseteq K(p_U)$.

Let

$$\underline{\Pi_U} = \max_{p_U} \; p_U \sum_{\Delta(\theta) \geq p_U} \mu(\theta) \tag{1}$$

be the largest profit the uninformed firm can secure from selling to its natural turf, and $\underline{p_U}$ the largest maximizer. As $\sum_{\Delta(\theta)\geq p_U} \mu(\theta)$ remains constant between two values of $\Delta(\theta)$, $\underline{p_U}$ is a value of $\Delta(\theta)$ on the support of μ. I write $m(p_U) = \underline{\Pi_U}/p_U$ the mass required to attain the profit lower bound at price p_U.

Perfect segmentation. Under perfect segmentation, every type is placed into its own segment, and the informed firm charges $\max\{w_I(\theta, p_U), 0\}$ to every type and sells its product to every consumer with a strictly positive WTP. Since $\theta_I > 0$, the types with $w_I(\theta, p_U) \leq 0$ are exactly those in $NT(p_U)$, so the uninformed firm sells exactly to $NT(p_U)$ and earns $p_U \mu\big(NT(p_U)\big)$. Its best responses are therefore the maximizers in (1). Keeping the segmentation fixed, the informed firm's profit increases in each segment at price p_U because $w_I(\theta, \cdot)$ is nondecreasing. Therefore, the informed firm's preferred equilibrium selects the largest price p_U. Perfect segmentation induces the price $\underline{p_U}$ and gives the uninformed firm the profit $\underline{\Pi_U}$.

Allocations. For $p_U > 0$, an *allocation at* p_U is a function $x : \Theta \to [0,1]$ such that $x(\theta) \leq \mu(\theta)$ for all θ, $x(\theta) = 0$ for $\theta \notin K(p_U)$, and $\sum_\theta x(\theta) \geq m(p_U)$. For a segmentation σ with equilibrium prices (p_I^σ, p_U^σ), I define an *equilibrium allocation* x^σ as the mass of type θ that buys from the uninformed firm, i.e.

$$x^\sigma(\theta) = \sum_{s \in S} \sigma(\theta, s)\, B_U\big(\theta, p_I^\sigma(s), p_U^\sigma\big)$$

I refer to consumers buying from the uninformed (informed) firm as uninformed (informed) firm's consumers. An allocation is *U-sorted* if there is $\hat{\delta}$ such that $x(\theta) = \mu(\theta)$ whenever $\Delta(\theta) > \hat{\delta}$ and $x(\theta) = 0$ whenever $\Delta(\theta) < \hat{\delta}$. A segmentation is U-sorted when its equilibrium allocation x^σ is.

Cheapest allocation. For $p_U > 0$ with $\mu\big(K(p_U)\big) \geq m(p_U)$, let x_{p_U} be the cheapest allocation at p_U such that its total mass is $m(p_U)$ and

$$x_{p_U}(\theta) = \begin{cases} \mu(\theta) & \text{if } \Delta(\theta) > \delta_{p_U} \text{ and } \theta \in K(p_U) \\ [0, \mu(\theta)] & \text{if } \Delta(\theta) = \delta_{p_U} \text{ and } \theta \in K(p_U) \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where δ_{p_U} is the largest value d such that $\mu\big(\{\theta \in K(p_U) : \Delta(\theta) \geq d\}\big) \geq m(p_U)$, which exists since the type space is finite. The choice within the tier $\{\theta : \Delta(\theta) = \delta_{p_U}\}$ does not affect the informed firm's profit, as all consumers have the same WTP.

Let

$$\Pi^M(p_U) = \sum_\theta \big(\mu(\theta) - x_{p_U}(\theta)\big)\, w_I(\theta, p_U)$$

be the profit under the cheapest allocation x_{p_U}.

Remark. If $\mu\big(\{\theta : \Delta(\theta) > 0\}\big) = 0$, the uninformed firm cannot earn a positive profit

from selling to its natural turf, and the informed firm optimally opts for perfect segmentation under which the uninformed firm is indifferent among all prices. In the informed firm's preferred equilibrium, the uninformed firm posts $p_U = v_K$. Since $\theta_U \leq v_K$ for every type, the informed firm charges θ_I in each segment and extracts the whole surplus $\sum_\theta \mu(\theta)\,\theta_I$, the largest profit it can earn. Every optimal segmentation must therefore lead every consumer to buy from the informed firm at price θ_I, so it is covered, I-extractive, and U-sorted. Since no consumer values either product above v_K, the price v_K is weakly higher than the uninformed firm's price under no-segmentation.

I assume for the proofs that $\mu\big(\{\theta : \Delta(\theta) > 0\}\big) > 0$, and I now introduce the following Lemmas essential for the proof of the theorems and corollaries.

Lemma 1 (U's profit lower bound)**.** For any segmentation σ and any $p_U > 0$, every consumer in $NT(p_U)$ buys from the uninformed firm when it posts p_U. The uninformed firm, therefore, earns at least $\underline{\Pi_U}$ in equilibrium, and x^σ is an allocation at p_U^σ that contains its whole natural turf: $x^\sigma(\theta) = \mu(\theta)$ for all $\theta \in NT(p_U^\sigma)$.

Proof. If $\theta \in NT(p_U)$, then $\theta_U \geq p_U$ and $w_I(\theta, p_U) \leq 0$, so θ never buys from the informed firm and buys from the uninformed firm. Posting $\underline{p_U}$ therefore yields at least $\underline{\Pi_U} > 0$, so $p_U^\sigma > 0$ and $p_U^\sigma \sum_\theta x^\sigma(\theta) \geq \underline{\Pi_U}$, that is, $\sum_\theta x^\sigma(\theta) \geq m(p_U^\sigma)$. Finally, only consumers with $\theta_U \geq p_U^\sigma$ can buy from the uninformed firm, so $x^\sigma(\theta) = 0$ for $\theta \notin K(p_U^\sigma)$. □

Lemma 2 (I's profit upper bound)**.** Let $p_U > 0$ with $\mu\big(K(p_U)\big) \geq m(p_U)$.

1. $x_{p_U}(\theta) = \mu(\theta)$ for every $\theta \in NT(p_U)$, and among all allocations at p_U, the cheapest allocation x_{p_U} minimizes $\sum_\theta x(\theta)\, g\big(w_I(\theta, p_U)\big)$ for every nondecreasing function $g \geq 0$.

2. Every segmentation σ with $p_U^\sigma = p_U$ satisfies

$$\Pi_I(\sigma) \;\leq\; \sum_\theta \big(\mu(\theta) - x^\sigma(\theta)\big)\, w_I(\theta, p_U) \;\leq\; \Pi^M(p_U)$$

The first inequality holds with equality if and only if σ is covered and I-extractive. The second holds with equality if and only if x^σ is a cheapest allocation.

3. Let $p_U \geq \underline{p_U}$. If some U-sorted allocation at p_U exists, then x_{p_U} is U-sorted, that is, every type with $\Delta(\theta) > \delta_{p_U}$ lies in $K(p_U)$, and $\delta_{p_U} \geq \underline{p_U} > 0$.

Proof. (i) Since $p_U\, \mu\big(NT(p_U)\big) \leq \underline{\Pi_U}$, we have $\mu\big(NT(p_U)\big) \leq m(p_U)$. The set $NT(p_U) = \{\theta : \Delta(\theta) \geq p_U\}$ lies in $K(p_U)$ and consists of the types with the highest $\Delta(\theta)$, so x_{p_U} contains it entirely. On $K(p_U)$, $w_I(\theta, p_U) = p_U - \Delta(\theta)$. Hence x_{p_U} places mass $m(p_U)$ on the lowest WTPs in $K(p_U)$. Every allocation at p_U is supported on $K(p_U)$ and has mass at least $m(p_U)$, so the claim follows because g is nondecreasing and nonnegative.

(ii) *First inequality.* A consumer who buys from the informed firm in segment s has $w_I(\theta, p_U) \geq p_I^\sigma(s) \geq 0$, so the informed firm earns at most $w_I(\theta, p_U)$ from him, with equality if and only if he pays his WTP. A consumer who buys from neither firm has $\theta_U < p_U$, so his WTP is $\theta_I > 0$. By Lemma 1, $w_I(\theta, p_U) > 0$ whenever $x^\sigma(\theta) < \mu(\theta)$. Summing gives the inequality. It holds with equality if and only if every consumer of the informed firm pays his WTP and every consumer buys one product, i.e., if and only if σ is I-extractive and covered.

Second inequality. By Lemma 1 and (i), x^σ and x_{p_U} both equal μ on $NT(p_U)$, which is exactly the set of types with $w_I(\theta, p_U) \leq 0$. The inequality is therefore equivalent to $\sum_\theta x^\sigma(\theta) \max\{w_I(\theta, p_U), 0\} \geq \sum_\theta x_{p_U}(\theta) \max\{w_I(\theta, p_U), 0\}$, which is (i) with $g(w) = \max\{w, 0\}$.

If x^σ is not a cheapest allocation, the inequality is strict. There are two cases.

- If x^σ has mass above $m(p_U)$, it puts excess mass on types outside $NT(p_U)$, which have $w_I(\theta, p_U) > 0$. Removing that excess strictly lowers the sum.

- Otherwise, there exists a type θ' with $x^\sigma(\theta') > 0$ and $\theta \in K(p_U)$ with $\Delta(\theta) > \Delta(\theta')$ and $x^\sigma(\theta) < \mu(\theta)$. Then $\theta' \notin NT(p_U)$, since otherwise $\theta \in NT(p_U)$ and $x^\sigma(\theta) = \mu(\theta)$. So $w_I(\theta', p_U) > \max\{w_I(\theta, p_U), 0\}$, and shifting mass from θ' to θ strictly lowers the sum.

(iii) Let x be a U-sorted allocation at p_U with threshold $\hat{\delta}$. Every type with $\Delta(\theta) > \hat{\delta}$ is served by x, so it lies in $K(p_U)$. Since x has mass at least $m(p_U)$ on $K(p_U) \cap \{\theta : \Delta(\theta) \geq \hat{\delta}\}$, the cutoff of x_{p_U} satisfies $\delta_{p_U} \geq \hat{\delta}$, so every type with $\Delta(\theta) > \delta_{p_U}$ lies in $K(p_U)$. Then x_{p_U} serves all these types and some type at δ_{p_U}, so $\mu\big(\{\theta : \Delta(\theta) > \delta_{p_U}\}\big) < m(p_U) \leq m(\underline{p_U}) = \mu\big(\{\theta : \Delta(\theta) \geq \underline{p_U}\}\big)$. Hence some type has $\underline{p_U} \leq \Delta(\theta) \leq \delta_{p_U}$, and $\delta_{p_U} \geq \underline{p_U}$. □

Lemma 3 (Capacity)**.** Consider some segmentation σ with $p_U = p_U^\sigma$. Then $\mu\big(K(p_U)\big) \geq m(p_U)$ and the cheapest allocation x_{p_U} satisfies

$$q \sum_\theta x_{p_U}(\theta) \mathbb{1}\{w_I(\theta, p_U) \geq q\} \leq \sum_\theta (\mu(\theta) - x_{p_U}(\theta)) \max\{w_I(\theta, p_U) - q, 0\} \tag{CAP}$$

for all $q > 0$.[5]

Proof. The first claim is trivial, and follows from Lemma 1. For the second claim, consider a segment s with price $p_I = p_I^\sigma(s)$ and take a $q > 0$. Let N_I^s denote the mass of the informed firm's consumers in s and $N_U^s(q)$ the mass of the uninformed firm's consumers in s with $w_I(\theta, p_U) \geq q$.

1. If $q < p_I$, then optimality of the informed firm's price is such that $p_I N_I^s \geq q(N_I^s + N_U^s(q))$ and therefore $qN_U^s(q) \leq (p_I - q)N_I^s \leq \sum_\theta \sigma(\theta, s) \mathbb{1}\{w_I(\theta, p_U) \geq p_I\}(w_I(\theta, p_U) - q)$

2. If $q \geq p_I$, then $N_U^s(q) = 0$ because every uninformed firm's consumer has $w_I(\theta, p_U) < p_I \leq q$ or $w_I(\theta, p_U) = p_I = 0 < q$.

Summing over segments yields

$$q \sum_\theta x^\sigma(\theta) \mathbb{1}\{w_I(\theta, p_U) \geq q\} \leq \sum_\theta \big(\mu(\theta) - x^\sigma(\theta)\big) \max\{w_I(\theta, p_U) - q, 0\}$$

By Lemma 2(i), applied with $g(w) = \mathbb{1}\{w \geq q\}$ and $g(w) = \max\{w - q, 0\}$, replacing x^σ by x_{p_U} weakly lowers the left-hand side of the summed inequality and weakly raises its right-hand side. This yields (CAP). □

[5] (CAP) is the discrete version of (AIC) from Elliott et al. (2026).

Lemma 4 (Implementation)**.** Let $p_U \geq \underline{p_U}$ such that $\mu\big(K(p_U)\big) \geq m(p_U)$, x_{p_U} is U-sorted, and (CAP) holds. There exists a segmentation σ^{p_U} such that $p_U^{\sigma^{p_U}} \geq p_U$ and $\Pi_I(\sigma^{p_U}) \geq \Pi^M(p_U)$.

Proof. Throughout the proof, I write $w(\theta) = w_I(\theta, p_U)$. The informed firm's consumers are the mass $\mu(\theta) - x_{p_U}(\theta)$ of each type θ, and the uninformed firm's consumers are the mass $x_{p_U}(\theta)$. Since $x_{p_U}(\theta) = \mu(\theta)$ for every $\theta \in NT(p_U)$ by Lemma 2(i), every informed firm's consumer has $w(\theta) > 0$.

Construction. Let $y_1, \ldots, y_J$ be the distinct WTP levels of the informed firm's consumers, and let $a_j > 0$ be the informed firm's consumer mass at level y_j. Let $z_1 > z_2 > \cdots > z_L > 0$ be the distinct strictly positive WTP levels of the uninformed firm's consumers, and let $b_l > 0$ be the uninformed firm's consumer mass at level z_l. For $y_j > z_l$, define the *capacity*

$$\kappa_j(z_l) = a_j \, \frac{y_j - z_l}{z_l}$$

the largest mass of consumers with WTP z_l that can be pooled with the consumers with WTP y_j without giving the informed firm a strict incentive to poach them. The capacity is decreasing in z_l. In this notation, (CAP) evaluated at $q = z_l$ reads

$$\sum_{l' \leq l} b_{l'} \;\leq\; \sum_{j:\, y_j > z_l} \kappa_j(z_l) \qquad \text{for every } l$$

I construct masses $t_j^l \geq 0$ of the uninformed firm's consumers with WTP z_l to be pooled with the informed firm's consumers with WTP y_j, starting from the highest WTP. Set $T_j^0 = 0$ for all j. For $l = 1, \ldots, L$:

1. for each j with $y_j > z_l$, compute the slack $r_j^l = \kappa_j(z_l) - T_j^{l-1}$
2. allocate $t_j^l = b_l \, r_j^l \big/ \sum_{k:\, y_k > z_l} r_k^l$ to each such j, and $t_j^l = 0$ whenever $y_j \leq z_l$
3. update $T_j^l = T_j^{l-1} + t_j^l$

I show by induction on l that each step is well defined and that $T_j^l \leq \kappa_j(z_l)$ whenever $y_j > z_l$.

- *Slacks are weakly positive.* (CAP) leads the algorithm to work for $l = 1$. For $l > 1$, if $y_j > z_{l-1}$, the induction hypothesis leads to

$$\begin{aligned} T_j^{l-1} = T_j^{l-2} + t_j^{l-1} &\leq T_j^{l-2} + r_j^{l-1} \\ &= T_j^{l-2} + \kappa_j(z_{l-1}) - T_j^{l-2} = \kappa_j(z_{l-1}) \end{aligned}$$

  where the inequality holds because $\sum_{k:y_k > z_{l-1}} T_k^{l-2} = \sum_{l'<l-1} b_{l'}$ and (CAP) can be rewritten as

$$b_{l-1} \leq \underbrace{\sum_{k:y_k > z_{l-1}} \kappa_k(z_{l-1}) - \sum_{l'<l-1} b_{l'}}_{\sum_{k:y_k > z_{l-1}} r_k^{l-1}}$$

  Using the monotonicity of κ_j gives $T_j^{l-1} \leq \kappa_j(z_{l-1}) \leq \kappa_j(z_l)$. If $z_l < y_j \leq z_{l-1}$, then $T_j^{l-1} = 0$. In both cases $r_j^l \geq 0$.

- *Total slack covers level l.* Using (CAP) and the above statement

$$\sum_{k:y_k > z_l} r_k^l = \sum_{k:y_k > z_l} \kappa_k(z_l) - \sum_{k:y_k > z_l} T_k^{l-1} = \sum_{k:y_k > z_l} \kappa_k(z_l) - \sum_{l'<l} b_{l'} \geq b_l$$

Hence, step 2 is well defined, $t_j^l \leq r_j^l$, and therefore $T_j^l \leq \kappa_j(z_l)$. Moreover, $\sum_j t_j^l = b_l$, so every uninformed firm's consumer is placed.

Segment construction. For each type θ with WTP y_j and $\mu(\theta) - x_{p_U}(\theta) > 0$, I build one

segment s_θ with

$$\sigma^{p_U}(\theta', s_\theta) = \begin{cases} \mu(\theta) - x_{p_U}(\theta) & \text{if } \theta' = \theta \\ \dfrac{\mu(\theta) - x_{p_U}(\theta)}{a_j} \dfrac{x_{p_U}(\theta')}{b_{l(\theta')}} t_j^{l(\theta')} & \text{if } w(\theta') > 0 \text{ and } x_{p_U}(\theta') > 0 \\ 0 & \text{otherwise} \end{cases}$$

where $l(\theta')$ is the index such that $z_{l(\theta')} = w(\theta')$. I also create one segment s_0 containing all consumers with a weakly negative WTP,

$$\sigma^{p_U}(\theta, s_0) = \mu(\theta)\, \mathbb{1}\{w(\theta) \leq 0\}$$

Segment s_θ thus contains all informed firm's consumers of type θ, pooled with a share $\big(\mu(\theta) - x_{p_U}(\theta)\big)/a_j$ of the mass t_j^l of uninformed firm's consumers with WTP z_l, for every l. The segmentation is consistent. For the uninformed firm's consumers,

$$\sum_j \sum_{\theta:\, w(\theta) = y_j} \frac{\mu(\theta) - x_{p_U}(\theta)}{a_j} \frac{x_{p_U}(\theta')}{b_{l(\theta')}} t_j^{l(\theta')} = \frac{x_{p_U}(\theta')}{b_{l(\theta')}} \sum_j t_j^{l(\theta')} = x_{p_U}(\theta')$$

using $\sum_{\theta:\, w(\theta) = y_j} \big(\mu(\theta) - x_{p_U}(\theta)\big) = a_j$. Consistency is immediate for the informed firm's consumers and for the natural turf.

Optimality of $w(\theta)$ at p_U. In segment s_θ, the informed firm's consumers have WTP y_j, and all other consumers have a strictly lower WTP. Demand is constant on each interval $(z_{l+1}, z_l]$, so only the prices z_l with $z_l < y_j$ need to be checked. For each such l, the bound $T_j^l \leq \kappa_j(z_l)$ is equivalent to

$$z_l\big(a_j + T_j^l\big) \;\leq\; y_j\, a_j$$

and multiplying by $\big(\mu(\theta) - x_{p_U}(\theta)\big)/a_j$ gives the same comparison for the masses in s_θ.

Cutting the price to z_l is therefore not profitable, and under the informed firm's preferred equilibrium, it charges $w(\theta) = y_j$ in every segment s_θ. At p_U, the uninformed firm sells exactly to x_{p_U} and earns $p_U\, m(p_U) = \underline{\Pi_U}$, while the informed firm earns $\Pi^M(p_U)$.

Deviation to a lower price p'_U. Let $p'_U = p_U - \epsilon$ and consider a segment s_θ. The uninformed firm's consumers have $\theta_U \geq p_U$, so their WTP decreases by exactly ϵ, while the WTP of the informed firm's consumers of type θ decreases by at most ϵ. Cutting the price to $z_l - \epsilon$ attracts θ and the uninformed firm's consumers with WTP at least z_l at p_U. This is not profitable, because

$$(z_l - \epsilon)\big(a_j + T_j^l\big) = z_l\big(a_j + T_j^l\big) - \epsilon\big(a_j + T_j^l\big) \leq y_j a_j - \epsilon\big(a_j + T_j^l\big) \leq (y_j - \epsilon)\, a_j$$

where the first inequality was shown above, and the second uses $T_j^l \geq 0$ and is strict whenever $T_j^l > 0$, the only case in which the cut attracts any of the uninformed firm's consumers. Multiplying by $\big(\mu(\theta) - x_{p_U}(\theta)\big)/a_j$ gives the same comparison for the masses in s_θ. Keeping θ at the price $w_I(\theta, p'_U) \geq y_j - \epsilon$ earns at least $y_j - \epsilon$ per unit. Hence, the informed firm does not poach the uninformed firm's consumers and retains θ whenever $w_I(\theta, p'_U) > 0$. If $w_I(\theta, p'_U) \leq 0$, then $y_j - \epsilon \leq 0$, so every other consumer in s_θ has a strictly negative WTP, and $\theta \in NT(p'_U)$ buys from the uninformed firm. Consumers in s_0 keep a weakly negative WTP.

Since x_{p_U} is U-sorted, x_{p_U} and $NT(p'_U)$ are nested. If $p'_U > \delta_{p_U}$, then $NT(p'_U) \subseteq \{\theta : \Delta(\theta) > \delta_{p_U}\} \subseteq K(p_U)$, so $x_{p_U}(\theta) = \mu(\theta)$ for $\theta \in NT(p'_U)$. Otherwise, x_{p_U} is supported on $\{\theta : \Delta(\theta) \geq \delta_{p_U}\} \subseteq NT(p'_U)$. Hence a downward deviation yields

$$p'_U \sum_\theta \max\big\{x_{p_U}(\theta),\, \mu(\theta)\mathbb{1}\{\theta \in NT(p'_U)\}\big\} = \max\Big\{p'_U\, m(p_U),\ p'_U\, \mu\big(NT(p'_U)\big)\Big\} \ \leq\ \underline{\Pi_U}$$

which is not profitable.

Equilibrium. Under segmentation σ^{p_U}, the uninformed firm earns $\underline{\Pi_U}$ at p_U and at most $\underline{\Pi_U}$ at any lower price, so it has a best response $\hat{p}_U \geq p_U$. Holding the segmentation fixed, the informed firm's profit in each segment is nondecreasing in p_U, because $w_I(\theta, \cdot)$ is nondecreasing. The informed firm therefore earns at least $\Pi^M(p_U)$ when the uninformed firm posts $\hat{p}_U$. Hence the informed firm's preferred equilibrium gives $\Pi_I(\sigma^{p_U}) \geq \Pi^M(p_U)$, and it can be taken with $p_U^{\sigma^{p_U}} \geq p_U$. □

Lemma 5 (Strict monotonicity of Π^M)**.** Let $p_U < p'_U$ such that $\mu\big(K(p_U)\big) \geq m(p_U)$, $\mu\big(K(p'_U)\big) \geq m(p'_U)$ and $x_{p'_U}$ is U-sorted. Then $\Pi^M(p_U) < \Pi^M(p'_U)$.

Proof. Every type with $\Delta(\theta) > \delta_{p'_U}$ lies in $K(p'_U) \subseteq K(p_U)$. Taking the types of $K(p_U)$ in decreasing order of $\Delta(\theta)$ therefore exhausts every type θ such that $x_{p'_U}(\theta) = \mu(\theta)$ before reaching the tier $\{\theta : \Delta(\theta) = \delta_{p'_U}\}$. If $\delta_{p_U} = \delta_{p'_U}$, then types $\theta \in K(p'_U)$ are chosen in priority to be put within x_{p_U}. This choice does not affect $\Pi^M(p_U)$, because types in $\{\theta : \Delta(\theta) = \delta_{p_U}\}$ have the same WTP. Therefore, I have that $x_{p'_U}(\theta) \leq x_{p_U}(\theta)$ for all θ which holds because $m(p'_U) < m(p_U)$. Then

$$\Pi^M(p'_U) - \Pi^M(p_U) = \sum_\theta \big(\mu(\theta) - x_{p_U}(\theta)\big)\big[w_I(\theta, p'_U) - w_I(\theta, p_U)\big] + \sum_\theta \big(x_{p_U}(\theta) - x_{p'_U}(\theta)\big)\, w_I(\theta, p'_U)$$

The first sum is nonnegative because $w_I(\theta, \cdot)$ is nondecreasing. In the second sum, the released mass $m(p_U) - m(p'_U) > 0$ lies outside $NT(p'_U)$, because $x_{p'_U}(\theta) = \mu(\theta)$ for $\theta \in NT(p'_U)$ by Lemma 2(i). There, $w_I(\theta, p'_U) > 0$. Hence $\Pi^M(p_U) < \Pi^M(p'_U)$. □

Lemma 6 (Perfect segmentation floor)**.** Every optimal segmentation σ^* satisfies $p_U^{\sigma^*} \geq \underline{p_U}$ and $\Pi_I(\sigma^*) \geq \Pi^M(\underline{p_U})$.

Proof. Let $p_U^* = p_U^{\sigma^*}$. Since $NT(\underline{p_U}) \subseteq K(\underline{p_U})$ and $\mu\big(NT(\underline{p_U})\big) = m(\underline{p_U})$, Lemma 2(i) applies at $\underline{p_U}$, and $x_{\underline{p_U}}(\theta) = \mu(\theta)$ for all $\theta \in NT(\underline{p_U})$ and 0 elsewhere, which is U-sorted. All its

consumers have $w_I(\theta, \underline{p_U}) \leq 0$, so (CAP) holds at $\underline{p_U}$, and Lemma 4 gives

$$\Pi_I(\sigma^{\underline{p_U}}) \geq \Pi^M(\underline{p_U}) = \sum_\theta \mu(\theta) \max\{w_I(\theta, \underline{p_U}), 0\}$$

Suppose that $p_U^* < \underline{p_U}$. By Lemma 2(ii) and monotonicity of $w_I(\theta, p_U)$ in p_U, I have

$$\begin{aligned}\Pi_I(\sigma^*) \leq \sum_\theta \big(\mu(\theta) - x^{\sigma^*}(\theta)\big) \max\{w_I(\theta, p_U^*), 0\} &\leq \sum_\theta \big(\mu(\theta) - x^{\sigma^*}(\theta)\big) \max\{w_I(\theta, \underline{p_U}), 0\} \\ &= \Pi^M(\underline{p_U}) - \sum_\theta x^{\sigma^*}(\theta) \max\{w_I(\theta, \underline{p_U}), 0\}\end{aligned}$$

By Lemma 1, x^{σ^*} has mass at least $\underline{\Pi_U}/p_U^* > \underline{\Pi_U}/\underline{p_U} = \mu\big(NT(\underline{p_U})\big)$, so it puts positive mass on types outside $NT(\underline{p_U})$, which have a strictly positive WTP at $\underline{p_U}$. Hence the last sum is strictly positive, and $\Pi_I(\sigma^*) < \Pi^M(\underline{p_U}) \leq \Pi_I(\sigma^{\underline{p_U}})$, contradicting optimality. □

For the next Lemma, I write $CS_U(p_U) = \sum_\theta \mu(\theta) \max\{\theta_U - p_U, 0\}$.

Lemma 7 (Surplus envelope)**.** Let $\hat{p}_U > 0$ be such that $\mu\big(K(\hat{p}_U)\big) \geq m(\hat{p}_U)$ and the cheapest allocation $x_{\hat{p}_U}$ has a nonnegative cutoff, $\delta_{\hat{p}_U} \geq 0$. Then every segmentation σ with $p_U^\sigma \geq \hat{p}_U$ satisfies

$$CS(\sigma) \leq CS_U(\hat{p}_U) + \Pi^M(\hat{p}_U) - \Pi_I(\sigma)$$

Proof. Let $p_U = p_U^\sigma$. A consumer buying from the informed firm contributes θ_I to $CS(\sigma) + \Pi_I(\sigma)$. A consumer buying from the uninformed firm has $\theta_U \geq p_U$ and contributes $\theta_U - p_U = \theta_I - w_I(\theta, p_U)$. A non-buyer contributes $0 < \theta_I$. Hence

$$CS(\sigma) + \Pi_I(\sigma) \leq \sum_\theta \mu(\theta)\theta_I - \sum_\theta x^\sigma(\theta)\, w_I(\theta, p_U) \tag{3}$$

By Lemma 1, x^σ is an allocation at p_U with $x^\sigma(\theta) = \mu(\theta)$ for $\theta \in NT(p_U)$, and by Lemma 2(i), $x_{p_U}(\theta) = \mu(\theta)$ for the same θ. Since $NT(p_U)$ is exactly the set of types with $w_I(\theta, p_U) \leq 0$,

Lemma 2(i) gives

$$\sum_{\theta} x^{\sigma}(\theta)\, w_I(\theta, p_U) \;\geq\; \sum_{\theta} x_{p_U}(\theta)\, w_I(\theta, p_U) = \underline{\Pi_U} - \sum_{\theta} x_{p_U}(\theta)\Delta(\theta) \tag{4}$$

where the equality uses $w_I(\theta, p_U) = p_U - \Delta(\theta)$ on $K(p_U)$ and $p_U\, m(p_U) = \underline{\Pi_U}$.

Write $\hat{\delta} = \delta_{\hat{p}_U}$. Since x_{p_U} is supported on $K(p_U) \subseteq K(\hat{p}_U)$ and $\Delta(\theta) \leq \hat{\delta} + \max\{\Delta(\theta) - \hat{\delta}, 0\}$, then aggregating over all consumer masses in x_{p_U} gives

$$\begin{aligned}
\sum_{\theta} x_{p_U}(\theta)\Delta(\theta) &\leq \hat{\delta}\, m(p_U) + \sum_{\theta \in K(\hat{p}_U)} x_{p_U}(\theta) \max\{\Delta(\theta) - \hat{\delta}, 0\} \\
&\leq \hat{\delta}\, m(p_U) + \sum_{\theta \in K(\hat{p}_U):\, \Delta(\theta) > \hat{\delta}} \mu(\theta)\big(\Delta(\theta) - \hat{\delta}\big) \\
&= \sum_{\theta} x_{\hat{p}_U}(\theta)\Delta(\theta) - \hat{\delta}\big(m(\hat{p}_U) - m(p_U)\big) \\
&\leq \sum_{\theta} x_{\hat{p}_U}(\theta)\Delta(\theta)
\end{aligned} \tag{5}$$

where the equality holds because $x_{\hat{p}_U}(\theta) = \mu(\theta)$ for $\theta \in K(\hat{p}_U)$ with $\Delta(\theta) > \hat{\delta}$ and places its remaining mass at $\Delta(\theta) = \hat{\delta}$, so

$$\sum_{\theta} x_{\hat{p}_U}(\theta)\Delta(\theta) = \sum_{\theta \in K(\hat{p}_U):\, \Delta(\theta) > \hat{\delta}} \mu(\theta)\Delta(\theta) + \hat{\delta}\Big(m(\hat{p}_U) - \sum_{\theta \in K(\hat{p}_U):\, \Delta(\theta) > \hat{\delta}} \mu(\theta)\Big)$$

and the last inequality uses $m(p_U) \leq m(\hat{p}_U)$ and $\hat{\delta} \geq 0$.

Combining inequalities (3), (4), and (5), and using $\sum_{\theta} x_{\hat{p}_U}(\theta) w_I(\theta, \hat{p}_U) = \underline{\Pi_U} - \sum_{\theta} x_{\hat{p}_U}(\theta)\Delta(\theta)$,

$$\begin{aligned}
CS(\sigma) + \Pi_I(\sigma) &\leq \sum_{\theta} \mu(\theta)\theta_I - \sum_{\theta} x_{\hat{p}_U}(\theta)\, w_I(\theta, \hat{p}_U) \\
&= \sum_{\theta} \mu(\theta)\big(\theta_I - w_I(\theta, \hat{p}_U)\big) + \sum_{\theta} (\mu(\theta) - x_{\hat{p}_U}(\theta)) w_I(\theta, \hat{p}_U) \\
&= CS_U(\hat{p}_U) + \Pi^M(\hat{p}_U)
\end{aligned}$$

since $\theta_I - w_I(\theta, \hat{p}_U) = \max\{\theta_U - \hat{p}_U, 0\}$. □

A.1 Proof of Theorem 1

Let σ^* be optimal, and let $p_U^* = p_U^{\sigma^*}$.

Step 1: U-sortedness under Condition 1. Let $p_U \geq \underline{p_U}$ with $\mu\big(K(p_U)\big) \geq m(p_U)$. If $\theta \notin K(p_U)$ and $\Delta(\theta) > 0$, then $\Delta(\theta) \leq \Delta(\theta')$ for every $\theta' \in K(p_U)$, since otherwise Condition 1 would give $\theta_U \geq \theta'_U \geq p_U$. Let $x(\theta) = \mu(\theta)$ on $\{\theta : \theta \in K(p_U) \text{ and } \Delta(\theta) > 0\}$ and 0 elsewhere. Every type that x does not serve has either $\Delta(\theta) \leq 0$ or a $\Delta(\theta)$ below that of every served type, so x is U-sorted. Its mass is at least $m(p_U)$: either every type with $\Delta(\theta) \geq \underline{p_U}$ lies in $K(p_U)$, and these types have mass $m(\underline{p_U}) \geq m(p_U)$, or one of them does not, and then $K(p_U) \subseteq \{\theta : \Delta(\theta) > 0\}$, so $x(\theta) = \mu(\theta)$ for $\theta \in K(p_U)$. Since x is supported on $K(p_U)$, it is therefore an allocation at p_U, and Lemma 2(iii) shows that x_{p_U} is U-sorted with $\delta_{p_U} \geq \underline{p_U}$.

Step 2: Structure. By Lemma 6, $p_U^* \geq \underline{p_U}$, so $x_{p_U^*}$ is U-sorted by Step 1. By Lemma 3 applied to σ^*, $\mu\big(K(p_U^*)\big) \geq m(p_U^*)$ and $x_{p_U^*}$ satisfies (CAP), so Lemma 4 yields a segmentation that gives the informed firm at least $\Pi^M(p_U^*)$. By the optimality of σ^*, $\Pi^M(p_U^*) \leq \Pi_I(\sigma^*)$. By Lemma 2(ii), I also have

$$\Pi_I(\sigma^*) \leq \sum_\theta \big(\mu(\theta) - x^{\sigma^*}(\theta)\big) w_I(\theta, p_U^*) \leq \Pi^M(p_U^*)$$

Therefore, both inequalities hold with equality. Hence σ^* is covered and I-extractive, and x^{σ^*} is a cheapest allocation at p_U^*. Since every type with $\Delta(\theta) > \delta_{p_U^*}$ lies in $K(p_U^*)$ by Step 1, x^{σ^*} is U-sorted with threshold $\delta_{p_U^*}$: every consumer with $\Delta(\theta) > \delta_{p_U^*}$ buys from the uninformed firm and none with $\Delta(\theta) < \delta_{p_U^*}$ does. So σ^* is U-sorted.

Step 3: Price comparison. Lemma 6 shows that $p_U^* \geq \underline{p_U}$, the price induced by perfect segmentation. If $p_U^N < \underline{p_U}$, then $p_U^* > p_U^N$ also by Lemma 6.

If $p_U^N \geq \underline{p_U}$, Lemma 3 applied to no-segmentation and Step 1 show that $x_{p_U^N}$ is U-sorted and satisfies (CAP), so Lemma 4 yields a segmentation $\sigma^{p_U^N}$ with $\Pi_I(\sigma^{p_U^N}) \geq \Pi^M(p_U^N)$. If $p_U^* < p_U^N$, then Step 2 and Lemma 5 give

$$\Pi_I(\sigma^*) = \Pi^M(p_U^*) < \Pi^M(p_U^N) \leq \Pi_I(\sigma^{p_U^N})$$

contradicting the optimality of σ^*. Hence $p_U^* \geq p_U^N$.

A.2 Proof of Theorem 2

Let σ^* be an optimal segmentation with price p_U^*, and σ a U-sorted segmentation inducing price $p_U = p_U^\sigma$. Denote $\hat{p}_U = \max\{\underline{p_U}, p_U\}$.

Step 1: the informed firm can secure $\Pi^M(\hat{p}_U)$. By Lemma 6, $\Pi_I(\sigma^*) \geq \Pi^M(\underline{p_U})$, and the cutoff δ at $\underline{p_U}$ is $\underline{p_U} > 0$. If $p_U > \underline{p_U}$, Lemma 3 applied to σ gives $\mu\big(K(p_U)\big) \geq m(p_U)$ and (CAP) at p_U. By Lemma 1, x^σ is an allocation at p_U, and it is U-sorted by assumption, so Lemma 2(iii) shows that x_{p_U} is U-sorted with $\delta_{p_U} > 0$. Lemma 4 then yields a segmentation that gives the informed firm at least $\Pi^M(p_U)$. In both cases, $\Pi_I(\sigma^*) \geq \Pi^M(\hat{p}_U)$ and $\delta_{\hat{p}_U} > 0$.

Step 2: $p_U^* \geq \hat{p}_U$. Lemma 6 gives $p_U^* \geq \underline{p_U}$. If $p_U > \underline{p_U}$ and $p_U^* < p_U$, Lemmas 2(ii) and 5, the latter applicable because x_{p_U} is U-sorted by Step 1, give $\Pi_I(\sigma^*) \leq \Pi^M(p_U^*) < \Pi^M(p_U)$, which contradicts Step 1. Hence $p_U^* \geq \hat{p}_U$.

Step 3: Consumer surplus. The cutoff $\delta_{\hat{p}_U}$ is strictly positive by Step 1. By Step 2, Lemma 7 applies with $\hat{p}_U$, and together with Step 1 it gives

$$CS(\sigma^*) \leq CS_U(\hat{p}_U) + \Pi^M(\hat{p}_U) - \Pi_I(\sigma^*) \leq CS_U(\hat{p}_U) \leq CS_U(p_U)$$

Under segmentation σ, each consumer can always buy from the uninformed firm at p_U or

buy nothing, so his surplus is at least $\max\{\theta_U - p_U, 0\}$. Hence $CS_U(p_U) \leq CS(\sigma)$, and $CS(\sigma^*) \leq CS(\sigma)$.

A.3 Hotelling line

The arguments of Lemmas 1, 2 and 5 do not rely on the finiteness of the type space. With a continuum of types, segmentations are measures over types and segments, as in Elliott et al. (2026), and the capacity condition (CAP) has a continuous counterpart, their AIC condition, under which their construction yields a segmentation implementing the cheapest allocation.

Consider a unit mass of consumers indexed by $v \sim U[0,1]$. A consumer of type v values the uninformed firm's product at $\theta_U = V+v$ and the informed firm's product at $\theta_I = V+1-v$, for a parameter $V \geq 7/8$.[6] Preferences are perfectly anti-correlated: $\Delta(v) = \theta_U - \theta_I = 2v-1$, so consumers with $v < 1/2$ prefer the informed firm's product and consumers with $v > 1/2$ prefer the uninformed firm's product. Since both θ_U and $\Delta(v)$ increase with v, Condition 1 holds. Given a price p_U, the WTP of type v for the informed firm's product is

$$w(v, p_U) = \begin{cases} 1 - 2v + p_U & \text{if } v \geq p_U - V \\ V + 1 - v & \text{otherwise} \end{cases}$$

where the first case corresponds to consumers who can afford the uninformed firm's product.

No-segmentation. The uninformed firm posts its price first, and the informed firm best responds by solving

$$\max_{p_I} \; p_I \Pr(\theta_I - \theta_U \geq p_I - p_U)$$

[6] This assumption ensures that the market is covered under no-segmentation.

which gives $p_I = \frac{p_U+1}{2}$. Anticipating this response, the uninformed firm solves

$$\max_{p_U} \; p_U \Pr\left(\theta_U - \theta_I \geq p_U - \tfrac{p_U+1}{2}\right)$$

which gives $p_U^N = 1.5$ and $p_I^N = 1.25$. The informed firm serves $v \in [0, 0.625]$ and the uninformed firm serves $v \in (0.625, 1]$, as depicted in Figure 6. Profits are $\Pi_I = 0.78125$ and $\Pi_U = 0.5625$, consumer surplus is $CS = V - 0.609375$, and total welfare is $W = V + 0.734375$.

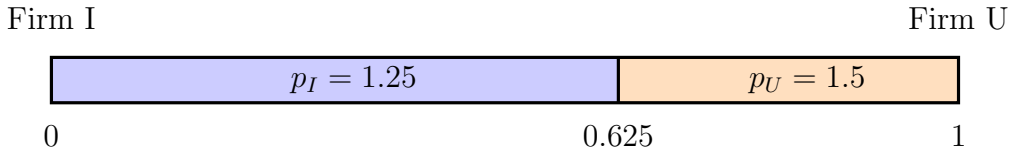


Figure 6: Pricing game equilibrium under no-segmentation

Perfect segmentation. Under perfect segmentation, the uninformed firm sells exactly to its natural turf and solves

$$\max_{p_U} \; p_U \Pr(\theta_U - \theta_I \geq p_U)$$

which gives $\underline{p_U} = 1/2$ and $\underline{\Pi_U} = 1/8$. Its natural turf is $NT(\underline{p_U}) = [3/4, 1]$, and the informed firm charges every other consumer his full WTP, $w(v, \underline{p_U}) = 1.5 - 2v$, as depicted in Figure 7. Profits are $\Pi_I = 0.5625$ and $\Pi_U = 0.125$, consumer surplus is $CS = V$, and total welfare is $W = V + 0.6875$. The outcome reflects the classic effect of price discrimination in competition: threatened by the informed firm's ability to poach its consumers, the uninformed firm lowers its price. Although the informed firm serves more consumers than under no-segmentation, it faces fiercer competition, and consumers are the main beneficiaries. Consumers with $v < 1/8$ who strongly prefer the informed firm's product nevertheless lose surplus relative to the no-segmentation case.

Optimal segmentation. Since Condition 1 holds, the optimal segmentation is covered, I-

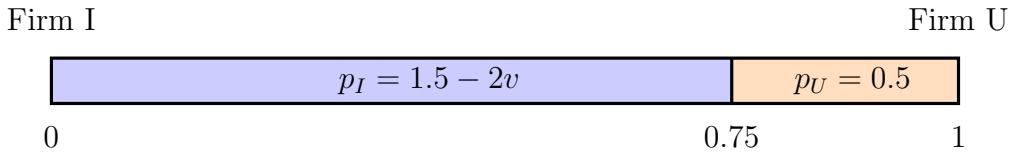


Figure 7: Pricing game equilibrium under perfect segmentation

extractive and U-sorted. At a price p_U, the uninformed firm must sell to a mass $m(p_U) = \frac{1}{8p_U}$ to earn its profit lower bound, and the cheapest consumers to cede are those with the highest $\Delta(v)$, namely $v \in \left[1 - \frac{1}{8p_U}, 1\right]$. These consumers must be able to afford the uninformed firm's product. Since consumers who can afford the uninformed firm's product are in $[p_U - V, 1]$, this requires $\mu\big([p_U - V, 1]\big) \geq \underline{\Pi_U}/p_U$, that is, $V + 1 - \frac{1}{8p_U} \geq p_U$, or

$$p_U \;\leq\; \bar{p} \;:=\; \frac{(V+1) + \sqrt{(V+1)^2 - \frac{1}{2}}}{2}$$

By Lemma 1, there exists no segmentation that can induce a price above $\bar{p}$, since the uninformed firm could not sell to enough consumers to earn $\underline{\Pi_U}$. By Lemma 2, every segmentation inducing a price $p_U \leq \bar{p}$ gives the informed firm at most $\Pi^M(p_U)$, which is strictly increasing in p_U by Lemma 5: a higher price raises every WTP for the informed firm's product and reduces the mass the informed firm must cede. The optimal segmentation, therefore, induces $p_U^* = \bar{p}$, provided the cheapest allocation at $\bar{p}$ can be implemented. I show below that it satisfies the AIC condition of Elliott et al. (2026), the continuous counterpart of (CAP), and construct the corresponding segmentation. The uninformed firm has no profitable downward deviation by the argument of Lemma 4, since the cheapest allocation is U-sorted. It has no profitable upward deviation either: at any $p_U > \bar{p}$, fewer than $\frac{\Pi_U}{p_U}$ consumers can afford its product, so it would earn less than $\underline{\Pi_U}$.

At p_U^*, the affordability constraint binds: the lowest ceded consumer, $\underline{v} = 1 - \frac{1}{8p_U^*} = p_U^* - V$, values the uninformed firm's product exactly at p_U^*. Every consumer $v < \underline{v}$ cannot afford the uninformed firm's product. Therefore, his WTP for the informed firm's product is his

full valuation, and the informed firm charges $V + 1 - v$. Consumers $v \geq \underline{v}$ buy from the uninformed firm at p_U^* and obtain $v - \underline{v}$. The pricing game equilibrium is depicted in Figure 8. Profits are

$$\Pi_I = V + \frac{1}{2} - \frac{V}{8p_U^*} - \frac{1}{128(p_U^*)^2} \qquad \text{and} \qquad \Pi_U = \frac{1}{8}$$

consumer surplus is $CS = \frac{1}{128(p_U^*)^2}$, and total welfare is $W = V + \frac{5}{8} - \frac{V}{8p_U^*}$. As predicted by Corollary 1, every consumer obtains exactly what the uninformed firm's offer is worth to him, $\max\{\theta_U - p_U^*, 0\}$, so every consumer is worse off than under both benchmarks. Consumers buying from the informed firm obtain nothing, and the uninformed firm's consumer surplus vanishes as V grows.

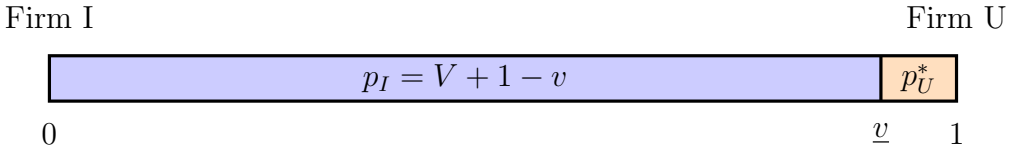


Figure 8: Pricing game equilibrium under the optimal segmentation

Comparison. In all three regimes, the inefficiency takes the form of consumers $v \in \left(\frac{1}{2}, \bar{v}\right)$ buying from the informed firm, although they prefer the uninformed firm's product, where $\bar{v}$ denotes the market split. Since each such consumer loses $\theta_U - \theta_I = 2v - 1$, the deadweight loss is $\left(\bar{v} - \frac{1}{2}\right)^2$. It equals $\frac{1}{64}$ under no-segmentation, $\frac{1}{16}$ under perfect segmentation, and $\left(\frac{1}{2} - \frac{1}{8p_U^*}\right)^2$ under the optimal segmentation, which tends to $\frac{1}{4}$ as V grows: the optimal segmentation is the least efficient of the three, since the informed firm ends up serving almost the entire line. Table 3 reports the surplus decomposition at $V = 10$, where $p_U^* \approx 10.99$, far above both $p_U^N = 1.5$ and $\underline{p_U} = 0.5$. The optimal segmentation transfers almost all consumer surplus to the informed firm.

	Π_I	Π_U	CS	W	DWL
No-segmentation	0.7813	0.5625	9.3906	10.7344	0.0156
Perfect segmentation	0.5625	0.1250	10.0000	10.6875	0.0625
Optimal segmentation	10.3862	0.1250	0.0001	10.5113	0.2388

Table 3: Surplus decomposition under the three regimes ($V = 10$)

Capacity condition at p_U^*

Throughout, write $w(v) = w(v, p_U^*)$ and let

$$\underline{w} := w(1) = p_U^* - 1, \qquad \overline{w} := w(\underline{v}) = p_U^* - 1 + \frac{2}{8p_U^*}$$

denote the lowest and highest WTP among the uninformed firm's consumers. Since the affordability constraint binds at p_U^*, $V = p_U^* - 1 + \frac{1}{8p_U^*}$, so that $\overline{w} = 2V + 1 - p_U^*$ and $p_U^* - V = \underline{v}$. The informed firm's consumers $v < \underline{v}$ lie on the branch $w(v) = V + 1 - v$, with density 1 in WTP space, and the uninformed firm's consumers lie on the branch $w(v) = 1 - 2v + p_U^*$, with density 1/2 in WTP space. The continuous counterpart of (CAP), the AIC condition of Elliott et al. (2026), requires for every $q > 0$

$$\int_0^{\underline{v}} \max\{w(v) - q, 0\}\, dv \;\geq\; q \int_{\underline{v}}^1 \mathbb{1}\{w(v) \geq q\}\, dv$$

I distinguish three cases.

1. When $q > \overline{w}$, no uninformed firm's consumer has a WTP above q, so the right-hand side equals 0 and the condition holds.

2. When $q < \underline{w}$, every uninformed firm's consumer has a WTP above q, so the right-hand side equals $q/(8p_U^*)$ and is increasing in q, while the left-hand side is decreasing in q. If the condition holds at $q = \underline{w}$, it therefore holds for all $q < \underline{w}$.

3. When $\underline{w} \leq q \leq \overline{w}$, the condition must be checked directly.

In the third case, every informed firm's consumer has a WTP above q, so the left-hand side equals

$$\int_0^{\underline{v}} \big(w(v) - q\big)\, dv = \frac{\big(w(0) - w(\underline{v})\big)\big(w(0) + w(\underline{v}) - 2q\big)}{2} = \frac{(p_U^* - V)\big(3V + 2 - p_U^* - 2q\big)}{2}$$

and the right-hand side equals

$$q \int_q^{\overline{w}} \tfrac{1}{2}\, dw = \frac{q\big(\overline{w} - q\big)}{2} = \frac{q\big(2V + 1 - p_U^* - q\big)}{2}$$

Denoting by $D(q)$ the left-hand side minus the right-hand side, and using $V = p_U^* - 1 + \frac{1}{8p_U^*}$,

$$D(q) = \frac{q^2}{2} - \frac{p_U^* + 1}{2}\, q - \frac{(V - p_U^*)^2}{2} + p_U^* - \frac{1}{8} + \frac{1}{8p_U^*} - \frac{1}{64(p_U^*)^2}$$

Since D is convex in q, its minimum is attained at $\hat{q} := \frac{p_U^* + 1}{2}$. As $\hat{q} \geq \underline{w}$ if and only if $p_U^* \leq 3$, it suffices to check $D(\hat{q}) \geq 0$ if $p_U^* \leq 3$, and $D(\underline{w}) \geq 0$ if $p_U^* > 3$.

Case $p_U^* > 3$. Using $\frac{1}{2}\underline{w}^2 - \frac{1}{2}(p_U^* + 1)\underline{w} = 1 - p_U^*$ and $p_U^* - V = 1 - \frac{1}{8p_U^*}$,

$$D(\underline{w}) = -\frac{(V - p_U^*)^2}{2} + \frac{56(p_U^*)^2 + 8p_U^* - 1}{64(p_U^*)^2} = \frac{(4p_U^* + 3)(12p_U^* - 1)}{128(p_U^*)^2} \;\geq\; 0$$

Case $p_U^* \leq 3$. Using $-\frac{(V - p_U^*)^2}{2} = -\frac{1}{2} + \frac{1}{8p_U^*} - \frac{1}{128(p_U^*)^2}$,

$$D(\hat{q}) = -\frac{(p_U^*)^2}{8} + \frac{3p_U^*}{4} - \frac{3}{4} + \frac{1}{4p_U^*} - \frac{3}{128(p_U^*)^2} = -\frac{(2p_U^* - 1)^2\big(4(p_U^*)^2 - 20p_U^* + 3\big)}{128(p_U^*)^2}$$

which is nonnegative whenever $\frac{5 - \sqrt{22}}{2} \leq p_U^* \leq \frac{5 + \sqrt{22}}{2}$, and hence for $p_U^* \leq 3$, since $V \geq 7/8$ implies $p_U^* > 1.8$. The capacity condition thus holds in all cases.

Construction of the optimal segmentation

Following Elliott et al. (2026), define the capacity of the informed firm's consumers with WTP z at level q as

$$G(q \mid z) = \frac{z - q}{q}$$

since these consumers have density 1, and, for $q \geq \overline{w}$, the overall capacity

$$H(q) = \int_q^{V+1} \frac{z - q}{q}\, dz = \frac{(V + 1 - q)^2}{2q}$$

The mass of uninformed firm's consumers with a WTP greater than $c \in [\underline{w}, \overline{w}]$ is $Q(c) = \frac{\overline{w}-c}{2}$. Since H is decreasing and $H(\overline{w}) > \frac{1}{8p_U^*} = \max_c Q(c)$, the equation $H(\gamma(c)) = Q(c)$ defines a cutoff $\gamma(c) > \overline{w}$, above which the capacity is just enough to accommodate all uninformed firm's consumers with a WTP above c. With $R = 2(V + 1) + \overline{w} - c$, it solves $(V + 1)^2 - R\gamma + \gamma^2 = 0$, so

$$\gamma(c) = \frac{R - \sqrt{(\overline{w} - c)\big(R + 2(V + 1)\big)}}{2}$$

which is well defined and below $V + 1$ since $R - 2(V + 1) = \overline{w} - c \geq 0$. An uninformed firm's consumer with WTP c is pooled with informed firm's consumers with WTP z according to the density

$$\psi(z \mid c) = \left.\frac{\partial_q G(q \mid z)}{\partial_q H(q)}\right|_{q=\gamma(c)} = \frac{2z}{(V + 1)^2 - \gamma(c)^2} \qquad \text{for } z > \gamma(c)$$

and $\psi(z \mid c) = 0$ otherwise. The segmentation is consistent, since $\int_{\gamma(c)}^{V+1} \psi(z \mid c)\, dz = 1$.

It remains to verify that the informed firm never wishes to poach. Fix a segment in which the informed firm's consumers have WTP z, and a deviation price $q < z$. If $q > \overline{w}$, no uninformed firm's consumer has a WTP above q, and the deviation is not profitable.

Otherwise, let $c_0 = \max\{\underline{w}, q\}$ and change variables to $u = \gamma(c)$. Inverting $H(u) = Q(c)$ gives $c(u) = \overline{w} - \frac{(V+1-u)^2}{u}$, so that $\frac{1}{2}\,dc = \frac{(V+1)^2-u^2}{2u^2}\,du$ and $\psi(z \,|\, c)\,\frac{1}{2}\,dc = \frac{z}{u^2}\,du$. The mass of uninformed firm's consumers with a WTP above q in the segment, per unit mass of the informed firm's consumers, is therefore

$$\int_{\gamma(c_0)}^{z} \frac{z}{u^2}\,du = \frac{z-\gamma(c_0)}{\gamma(c_0)} = G\big(\gamma(c_0) \,|\, z\big) \;\leq\; G(q \,|\, z)$$

since $\gamma(c_0) \geq c_0 \geq q$ and $G(\cdot \,|\, z)$ is decreasing. Cutting the price to q is therefore not profitable, and the informed firm charges z in every segment.